

When Witness Search Becomes Address Construction

A guarded bridge note between Wolfram's ruliological P-vs-NP microcosms and τ -admissible witness construction — internal-fragment scope, no classical P-vs-NP claim

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ABSTRACT

Stephen Wolfram's 2026 essay on P vs NP and ruliological computation makes computational difficulty empirically visible in finite computational microcosms: small Turing-machine classes, runtime outliers, isolate machines, nondeterministic speedups, everything-machine reachability, and computational irreducibility [27]. The essay's contribution is to catalogue, by exhaustive enumeration in the ruliological tradition, where finite-system difficulty actually lives. This Research Note reads that essay as an external bridge surface for a separate and bounded Panta Rhei question: when does search stop being blind search and become structure? The answer here is not a slogan about the classical Clay problem. It is the scoped τ -admissibility claim of the computation/admissibility spine: under finite interface width, witness search is treated as address construction rather than blind enumeration [8]. The two readings are parallel, not nested: Wolfram describes where difficulty resides in finite systems; the present note specifies a criterion under which, in a typed admissible fragment, search admits a constructive address-resolution alternative. The note does not claim that Wolfram supplies validation for Category τ , that the ruliad is Category τ , that the classical ZFC/Turing-machine P-vs-NP problem is resolved, or that the τ -admissibility predicate is itself definable in ZFC at any specified arithmetic complexity.

Keywords P vs NP · ruliology · computational irreducibility · witness search · address resolution · computational bi-square · tau-admissibility · Category τ

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1. INTRODUCTION: THE BRIDGE AND THE BOUNDARY

The P-vs-NP problem is famously easy to state and famously hard to locate. In its orthodox form it asks whether every decision problem whose positive instances can be verified in polynomial time can also be solved in polynomial time by a deterministic Turing machine. Yet that tidy statement leaves many structural choices in the background: what counts as a machine, what counts as a representation, what kind of nondeterministic choice is being granted, how observers inspect computations, and what kind of infinity is being used when asymptotic limits are taken.

Wolfram's ruliological essay is valuable because it refuses to leave all of that invisible. It does not try to settle the full question. Instead it asks what can be learned by explicitly enumerating finite computational microcosms: small one-sided Turing machines, the integer functions they compute, the runtime profiles they exhibit, the hard cases that appear inside fixed rule classes, and the ways nondeterministic branching changes reachable behavior. This is not orthodox proof theory, but it is not mere illustration either. It is empirical theoretical

computer science in the literal sense: finite rule universes are sampled, measured, and compared. The value of Wolfram's ruliological framing is that it makes computational difficulty visible without first forcing it into a single orthodox proof template.

The τ -framework reaches the same danger from another direction. It asks what has to be true before search can be called search at all. In the framework's computation/admissibility spine, P-vs-NP-style questions are native only at an operational layer where there are agents, steps, and an efficiency measure. Once that typing is in place, the decisive distinction is not simply deterministic versus nondeterministic. It is admissible versus inadmissible. A τ -admissible verifier has finite interface width: there is a finite primordial depth beyond which the computation sees nothing new. Under that condition, a witness is no longer a flat bitstring certificate. It is a canonical address whose per-prime components can be resolved and reassembled by the Chinese remainder structure. The slogan "search becomes structure" means exactly this: search becomes construction only where a finite address architecture has been earned.

CLAIM BOUNDARY

This is a guarded bridge note, not a proof note. Wolfram's essay is used as a public comparator for finite computational difficulty. The formal collapse introduced below is internal to τ -admissible witness problems, after finite interface width and address structure have been defined. This is an internal equality for τ -admissible witness problems. It does not settle orthodox P vs NP over unrestricted Turing-machine / ZFC encodings. The note does not treat the ruliad and Category τ as the same object.

The claim tiers are deliberately separated. Wolfram supplies the external comparator. The τ -admissible collapse is an internal framework claim. Any τ -native or physical-computation extension would add further assumptions. Any orthodox Clay/ZFC export would require a separate encoding, size measure, reduction, and proof-system bridge.

The contribution of this note is a disciplined comparison. Section 2 reconstructs the Wolfram anchor in its own terms. Sections 3–7 reconstruct the τ computation source spine: computation layer, τ -Tower Machine, interface width, witness search, computational bi-square, and admissibility collapse. Sections 9–12 compare the two frameworks along four axes: finite exploration, nondeterminism, irreducibility, and maximal reachability. Section 14 then makes the orthodox boundary explicit. The closing section turns the comparison into a research program rather than a headline.

1.1 Anchor selection and reading method

Wolfram's essay is an unusually good anchor for this note because it has the right kind of distance from the τ -framework. It is close enough to share the core intuition that rule space, observers, and finite computational microcosms matter. It is distant enough that it cannot be mistaken for an internal source, a friendly paraphrase, or a validation surface. The essay is not written in the language of τ -addresses, primordial towers, CRT decomposition, or product-meet diagrams. It is therefore useful as an external comparator: it presses on the same nerve without using the same vocabulary.

The selection method is conservative. A paper or essay is a good anchor for this note only if it has three properties. First, it must discuss P-vs-NP difficulty through actual computational structure, not only through slogans about hard problems. Second, it must distinguish finite experiments from the full orthodox problem, so that the bridge does not begin with an overclaim. Third, it must expose a genuine contrast between search, reachability, and construction. Wolfram's essay satisfies all three. It builds finite machine microcosms; it repeatedly keeps the classical problem open; and it shows that nondeterministic reach can be studied without yet giving a deterministic construction method.

The reading method is correspondingly narrow. When this note says that Wolfram makes a difficulty visible, it means that the essay supplies a finite observable analogue: a runtime profile, a machine-class relative outlier, a multiway speedup,

or an everything-machine limit. When this note says that the τ spine answers a different question, it means that the internal source base imposes a typed admissibility criterion before claiming collapse. The comparison is not a two-column proof by analogy. It is a map of where two neighboring frameworks ask similar questions and where their answers stop being the same.

1.2 Three separations used throughout

To keep the paper readable, the following separations are used throughout.

1. *Reach is not construction.* A short path in a multiway graph is not yet a deterministic procedure for finding that path.
2. *Finite enumeration is not finite interface width.* A bounded machine class is an experimental microcosm; a finite-width verifier is an internal admissibility condition.
3. *Internal collapse is not orthodox export.* $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$ belongs to the τ -admissible fragment. The classical P versus NP problem over Turing machines and ZFC-style proof remains outside the claim of this note.

2. WOLFRAM'S FINITE RULIOLOGICAL EXPLORATION

The first obligation in a bridge note is fidelity. Wolfram's essay is not an appendix to our framework. It has its own problem, vocabulary, and method. The essay begins from the observation that P vs NP, in full form, is an infinite question: it quantifies over all possible inputs and all possible Turing machines. Wolfram's move is to ask finite subquestions inside that infinite problem. What happens if one fixes a small machine class and explicitly enumerates its behavior? What functions appear? Which machines compute them? What runtime profiles occur? When does adding states, colors, or nondeterministic alternatives change the answer? These are the finite subquestions Wolfram uses to replace the global problem with inspectable microcosms [27].

2.1 Finite machines as empirical theory

The machines in the essay are deliberately concrete. Inputs and outputs are read as integers; machines are specified by small state/color classes; halting and undefined behavior are tracked rather than swept away. This turns a famous asymptotic question into a family of finite maps: machine-class, computed function, runtime profile, and fastest-known representative. Within a fixed class, one can establish class-relative facts that are hard to obtain in ordinary complexity theory. For example, one can ask whether any machine of a given size computes a particular function faster than another, or whether a function is represented by only one machine in that finite class [27].

The word "relative" matters. A lower bound inside a fixed small machine class is not a lower bound for all Turing machines. Wolfram says this explicitly in practice, because larger machines can sometimes compute the same finite mapping by a lookup-style or more specialized rule. The result is not a

Table 1. Claim-tier map for the note. The tiers are separated near the front because the same words can otherwise sound much stronger than the manuscript intends.

Tier	What this note may say	What it must not imply
External Wolfram comparator	Wolfram’s ruliological essay is a valuable public anchor for finite computational microcosms, runtime wildness, nondeterministic reach, and the ruliad/everything-machine limit [27, 28].	It must not imply that Wolfram supplies validation for Category τ , proves any τ theorem, or resolves the classical P-vs-NP problem.
Internal τ claim	Inside the τ computation/admissibility spine, finite interface width and CRT witness decomposition support a scoped admissibility-collapse theorem [8].	It must not imply that the internal equality is the orthodox $P = NP$ theorem.
Orthodox P-vs-NP boundary	Any export to ordinary Turing-machine languages, bit-length polynomial time, reductions, and ZFC proof is a separate bridge problem.	It must not imply that the Clay problem is solved by this note.
Future export bridge	Later work may compare finite ruliological benchmarks with τ -interface-width diagnostics and explicit witness decompositions.	It must not imply that benchmark analogies count as validation, proof, or a universal solution recipe.

Table 2. The external anchor reconstructed as finite ruliological microcosms. The right column records the claim discipline used throughout this note.

Surface	What Wolfram studies	Use in this note	Guardrail
Finite machines	One-sided Turing machines with fixed state/color classes computing integer functions.	A concrete rule universe whose function and runtime structure can be inspected.	Not the same model as τ -Tower Machines.
Runtime outliers	Functions whose runtime profiles jump or contain dramatic hard inputs.	A finite view of why asymptotic behavior can be hard to infer.	Outliers are not universal lower bounds.
Isolate machines	Machines that are the unique or class-isolated computers of a function within a fixed finite class.	Bounded evidence for non-shortcut behavior.	Isolation is not τ -inadmissibility.
Nondeterminism	Multiway choice can shorten some computations, sometimes dramatically, but not uniformly.	A comparison surface for branch availability.	A branch is not yet deterministic witness construction.
Everything machine	A limiting finite-class object that includes all rule cases and reaches outputs by minimal paths.	A contrast for maximal nondeterministic reach.	Maximal reach is not τ construction.
Ruliad limit	The ruliad is the entangled limit of all possible computations, sampled by bounded observers.	A philosophical comparator for observer-bounded readout.	The ruliad is not Category τ .

naive claim that finite hard cases are hard forever. It is a careful demonstration that hardness is sensitive to the representation and rule universe in which it is measured [27].

For the present note, that class relativity is not a defect. It is the point. A finite computational universe makes visible what a purely asymptotic statement often hides: the cost of a computation depends on the grammar in which the computation is performed. A rule universe has state counts, color counts, tape conventions, input encodings, halting criteria, and output conventions. Change any of these and the observed microcosm can change. The τ -framework makes the analogous dependency explicit on its own side by refusing to speak about efficient search until the agent, register grammar, interface width, and witness type have been specified.

There is also an important sociological virtue here. The finite microcosms are inspectable. A reader can see particular machines, runtime tables, path graphs, and exception cases. That inspectability is why the essay is a better anchor for this

note than a purely philosophical essay about computational irreducibility would be. It gives the comparison an object: not merely “hardness exists,” but “here is how hardness appears when a finite rule class is made explicit.”

2.2 Outliers, isolates, and the failure of smoothness

One of the essay’s strongest findings is qualitative rather than merely enumerative. Runtime profiles in arbitrary small machines are not smooth objects. They can jump. They can contain exceptional inputs. They can depend on whether a machine halts, whether a larger machine is allowed, or whether one changes the amount of nondeterminism. For constructed algorithms, runtime often looks regular because the algorithm was designed to have a regular structure. For machines in the wild, the behavior is messier [27].

The “isolate machine” idea sharpens this point. If a machine is the only machine in a fixed finite class that computes a function, then its runtime profile becomes a class-relative lower bound for that function within that class. But even here

the conclusion remains bounded. The same function may be computed differently by a larger machine. The isolate shows that a finite computational microcosm can contain genuine non-shortcut structure; it does not license an unbounded asymptotic conclusion [27].

The isolate-machine surface is especially useful for a τ comparison because it separates two forms of uniqueness. In Wolfram's setting, uniqueness can mean uniqueness of a rule within a finite enumeration class. In the τ setting, uniqueness usually means uniqueness of a canonical address or normal representative after a construction grammar has been applied. These are not the same kind of uniqueness. The former is empirical and class-relative; the latter is structural and internal. The bridge is that both resist the idea that all computations are fungible black boxes. The boundary is that one uniqueness claim lives in an enumerated rule census, while the other lives in an address-resolution theorem.

Runtime outliers play a related role. A smooth complexity curve tempts one to extrapolate. An outlier interrupts the extrapolation and asks which hidden structure was missed. Wolfram's examples make that interruption visible at small scale. The τ -framework then asks a sharper question: is the outlier a sign of missing finite address structure, or is it merely a bad representation of an admissible computation? That question is not answered by the Wolfram essay, but the essay makes the question vivid [27].

2.3 Nondeterminism as reach

The P-vs-NP relevance becomes explicit when Wolfram introduces nondeterministic and multiway machines. A deterministic machine applies one rule at each step. A nondeterministic machine allows a choice among rules, creating a multiway graph of possible paths. In the essay's finite examples, this can shorten computations substantially. Some functions that take exponential time for a small deterministic machine can be reached much faster by a small nondeterministic machine. Other hard deterministic cases do not succumb to the smallest nondeterministic augmentation. That non-uniformity is one of the most useful facts for the present note [27].

Nondeterminism, in this setting, should be read as reach. It allows paths through a multiway graph that deterministic rules do not follow. But reach is not yet construction. A path may exist without giving a deterministic procedure for finding it. A multiway graph may contain a short route without making that route visible to a bounded deterministic observer. That distinction is the hinge on which the comparison with τ -admissibility turns [27].

This is also where finite experiments become philosophically sharp. Inside a small machine class, nondeterminism is not an abstraction floating above the model. It is a concrete addition of rule alternatives. One can draw the branch graph. One can ask which paths halt, which paths compute the same output, and which paths shorten the route. That concreteness prevents a common confusion: nondeterminism is not magic

knowledge. It is an expanded path structure. Whether that path structure can be converted into deterministic construction is a further question.

The τ note takes that further question as central. It asks not only whether there exists a short accepting branch, but whether the object certified by that branch has finite constructive coordinates. If the witness is still a black-box certificate, nondeterminism has supplied reach but not structure. If the witness decomposes into per-prime components and re-assembles canonically, the story changes. That is why the τ comparison cannot stop at the word "speedup." It has to ask what kind of thing has been sped up.

2.4 Everything-machine and ruliad limit

The essay then considers the limiting case of adding all conceivable rule choices inside a finite class: the "everything machine." This is a maximally nondeterministic object. It can reach outputs in the minimum number of steps structurally possible, and it functions as a fragment of the ruliad. In Wolfram's broader vocabulary, the ruliad is the entangled limit of all possible computations. It includes all possible computational paths, while actual observers sample bounded slices of it [27].

This is the deepest philosophical bridge to the Panta Rhei note, and also the easiest place to overclaim. The everything-machine/ruliad endpoint is about maximal reach. The τ computation spine is about admissible construction under a specific address grammar. Both make observer bounds and rule-space structure visible. But their primitives differ. The ruliad is a maximal computational ensemble; Category τ is a constructed framework with a primordial tower, CRT decomposition, finite interface width, and a product-meet proof architecture. The bridge is real because both ask what computation looks like when rule space is made explicit. The bridge is limited because maximal reach is not the same as finite admissible construction.

The everything-machine idea is nevertheless valuable because it provides a limiting counter-image to the τ collapse. In one direction, one tries to include more and more possible rules until reachability becomes maximal. In the other, one tries to identify the finite quotient through which a particular computation stabilizes. These are almost opposite gestures. The ruliological gesture is expansive: include more rules. The τ -admissible gesture is resolving: identify the finite structure that is actually needed. Their proximity lies in the fact that both reject a single fixed black-box machine as the final object of study. Their difference lies in the direction of explanation.

2.5 The essay's own boundary

The essay's conclusion is unusually useful for our purposes because it keeps the orthodox boundary intact. Wolfram states that finite ruliological subquestions do not directly resolve the full P-vs-NP problem. They give intuition and reveal difficulties: runtime wildness, halting subtleties, limit-order issues, and the role of computational irreducibility. That is

exactly the boundary this note must preserve. The Wolfram essay is an anchor because it makes the shape of the difficulty visible; it is not evidence that any external theorem has been settled [27].

This boundary is one reason the essay can be used responsibly. If the essay had claimed to solve P vs NP outright, the present note would first have to adjudicate that claim. Because it does not, the note can use the essay for what it does exceptionally well: it shows how quickly finite rule spaces become rich enough to resist simple summarization. That is a better bridge to τ -admissibility than a premature proof claim would be, because τ -admissibility is itself a boundary concept. It says: here is the fragment where construction is available; outside it, do not pretend that search has already been tamed.

3. THE τ COMPUTATION LAYER

The τ computation spine begins with a typing claim. P-vs-NP-style questions are not primitive at every layer of the framework. They require agents, steps, and an efficiency measure. Without an agent there is no searcher. Without steps there is no runtime. Without a relationship between input size and step count there is no polynomial-versus-exponential distinction. The computation layer says that these ingredients become native only at E_2 , where codes contain their own decoders and execution is a Code–Execution–Code cycle [8].

Categorical reading of the spine [τ -Effective]. The construction spine $E_0 \rightarrow E_1 \rightarrow E_2 \rightarrow E_3$ is, in the source-spine projection, a chain of *strata*: each E_i is a small category of typed objects (addresses at E_0 , dynamics at E_1 , self-referential codes at E_2 , observer-relative readout at E_3), and each arrow $E_i \rightarrow E_{i+1}$ is a forgetful functor in one direction with a constructive enrichment in the other. The categorical typing of the spine is source-spine material [8]; the substantive 100-step construction is given as an exhaustive expert-facing build in the PANTA RHEI CONSTRUCTION SPINE whitepaper (OSF 10.17605/OSF.I0/6GJ9K) [7], which the present note treats as the public-facing typed-spine reference. The bridge sections of the present note rely only on the property that E_2 has enough structure to host a self-referential decoder (so that “verifier,” “solver,” and “witness” have native meaning). Readers seeking the categorical machinery may consult the construction-spine whitepaper [7] and the Construction Spine public route [16]; readers willing to work with the spine as a sequence of typed strata can proceed without either. The present note does not assert that the spine is a tower of $(\infty, 1)$ -toposes, a chain of geometric morphisms, or an arithmetic fracture filtration; only that the four-stratum typing is what the source spine provides [3, 13].

Categorical typing of Adm_τ [τ -Effective]. The τ -admissibility predicate Adm_τ deployed throughout this note is, in the source-spine projection, a *propositional predicate on objects of E_2* — equivalently, a (proof-irrelevant) subobject classifier restricted to the τ -admissible fragment. In a realiz-

ability reading it is the analogue of the assembly-of-realizers admissibility condition in the effective topos [13]. The present bridge note does not commit to a stronger reading (sheaf condition, Lawvere–Tierney topology, internal modality); the framework-internal characterisation is deferred to [8].

This is not an orthodox complexity theorem. It is a framework typing statement. It says where the question becomes meaningful in the internal architecture. At E_0 , the framework has addresses and static structure, but not processes. At E_1 , it has dynamics, but not self-referential codes. At E_2 , it has agents whose decoder is itself representable as a code and whose execution produces new codes inside the same operational layer. Only there can one ask whether a search procedure is efficient.

This typing move matters for the comparison with Wolfram. Wolfram begins with finite rule systems and asks what their computed functions and runtimes look like. The τ source spine begins one level earlier: what structure must exist before a runtime question has a subject? Once the subject exists, the framework does not ask only whether a nondeterministic branch exists. It asks whether the witness lives inside a finite address architecture that a deterministic τ -agent can construct.

The resulting picture is less dramatic than a headline, but more useful as a research claim. The framework does not say that an arbitrary mathematical question becomes computationally easy once translated into τ -language. It says that the computational version of the question has a type. Before E_2 , there may be structures, transformations, and dynamics, but not yet a self-referential computational agent with an internal decoder. At E_2 , a program can act on codes and return codes inside the same typed layer. That is the first point at which the phrases “solver,” “verifier,” and “witness” have their native framework meaning.

This also prevents a false comparison with Wolfram’s small machines. Wolfram’s machines are deliberately minimal. They show how much behavior can be generated by tiny rule systems. The τ computation layer is not minimal in that sense; it is already a typed operational layer built on earlier address and enrichment structure. Its strength is not that it uses fewer primitives. Its strength is that it can say which primitives are doing the computational work. A finite Turing-machine enumeration finds difficulty by varying rules. The τ spine locates difficulty by asking whether the computation has earned finite readout.

4. τ -TOWER MACHINES AND INTERFACE WIDTH

The τ -Tower Machine is the concrete machine grammar used by the internal source spine. It resembles a register machine in having finite control and finitely many registers. It differs because the registers hold τ -addresses, not ordinary bitstrings. A program, its input data, its instruction set, and its output are all internal address-bearing objects. The observable transition of a machine reads a bounded window of this address structure,

Table 3. The τ computation/admissibility source spine used in this note. Book labels are source provenance; the manuscript frames the material as the τ computation spine.

Source locus	Registry IDs	Claim used here	Guardrail
Computation layer	III.D49, III.D50, III.R22	Agents, steps, and efficiency become native at E_2 .	Typing claim, not orthodox theorem.
τ -Tower Machine	III.D51, III.T30, III.D52	TTMs have finite control over τ -addressed registers and bounded observable transitions.	TTMs are not Wolfram machines.
Interface width	III.D53, III.D54, III.T31	τ -admissibility is finite stabilization in the primordial tower.	Applies inside Adm_τ .
Witness search	III.D55, III.P22, III.P23	Witnesses are τ -addresses decomposed by CRT into per-prime components.	Polynomial in primordial depth k , not automatically bit length.
Computational bi-square	III.D56, III.T32, III.T33	Product-Meet Collapse yields $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$.	Not classical $\text{P} = \text{NP}$.
No barrier / ZFC horizon	III.T34, III.D79, III.R46	The internal encoding gap is removed, while the ZFC bridge remains horizon-limited.	Provability/export language is scoped and partly conjectural.

which is why constant-width tableau language appears in the source spine.

This is not a cosmetic change of alphabet. In an ordinary Turing model, the tape is a flat string of symbols and the input-size metric is tied to that representation. In a TTM, the relevant data are organized by the primordial tower and by the finite quotient through which the computation stabilizes. The key invariant is *interface width*: the amount of primordial depth a computation must inspect before deeper levels contribute no new information.

Definition 4.1 (Interface-width reading [τ -Effective]). *For this note, a computation is treated as τ -admissible only when its verifier has finite interface width: there is a depth k_0 such that, for all relevant input sizes, the verifier’s observable behavior factors through the finite quotient at depth k_0 . The infinite tower then collapses, for that computation, to a finite computational horizon.*

The phrase “for that computation” is load-bearing. It blocks the most tempting overclaim. The source spine does not say that every arbitrary problem in every formal system magically has small interface width. It defines a class of admissible computations and proves the internal consequences inside that class. A function whose required depth grows with the input remains outside the admissible fragment until a finite-width factorization is supplied.

Interface width should therefore be read as a test, not a decoration. A candidate τ computation must answer concrete questions. What finite primordial quotient carries the relevant information? What does the verifier inspect? Which deeper coordinates are provably irrelevant to the accept/reject decision? How is the stability witnessed? If these questions cannot be answered, the computation has not entered the admissible fragment merely because it has been described in τ language.

The width condition also explains why the internal complexity parameter is not automatically ordinary input length. Primordial depth k controls a tower-indexed finite quotient.

Ordinary bit length controls the encoding size of a string in a Turing representation. Relating the two requires an export theorem or at least an explicit encoding policy. This note uses k -polynomial language only in the internal sense. It does not silently convert it into a classical polynomial-time statement.

Definition 4.2 (τ -admissible witness problem [τ -Effective]). *For this note, a τ -admissible witness problem is a decision problem presented to an E_2 -level τ -agent such that:*

1. *the verifier factors through a finite primordial quotient;*
2. *accepting witnesses are τ -addresses at that quotient;*
3. *the witness constraints decompose into finite local conditions on the relevant prime-indexed coordinates;*
4. *compatible local witnesses reassemble canonically by CRT; and*
5. *the verifier’s deeper tower data do not change the accept/reject decision.*

If any of these clauses is missing, the problem has not yet entered the admissible fragment used by the collapse claim.

Definition 4.3 (The classes $\tau\text{-P}_{\text{adm}}$ and $\tau\text{-NP}_{\text{adm}}$ [τ -Effective]). *$\tau\text{-P}_{\text{adm}}$ denotes the τ -admissible decision problems for which a deterministic τ -Tower Machine constructs the accepting address, when it exists, in time polynomial in the internal primordial depth. $\tau\text{-NP}_{\text{adm}}$ denotes the τ -admissible decision problems for which a finite-width τ -verifier checks an accepting address in polynomial internal depth. Both classes are already restricted by τ -admissibility; neither is the ordinary P or NP class of orthodox complexity theory.*

What kind of object $\tau\text{-P}_{\text{adm}}$ is [τ -Effective]. A standard complexity class in the sense of $\text{TIME}(f(n))$ requires a fixed input-size measure n together with a runtime bound $f(n)$. The present definitions of $\tau\text{-P}_{\text{adm}}$ and $\tau\text{-NP}_{\text{adm}}$ supply a runtime bound (in the internal depth parameter k) but *not* a fixed encoding of k as a function of orthodox bit length n . The relationship $k = k(n)$ is itself part of the bridge work, and is not asserted in this note: until $k = k(n)$ is pinned down for

a chosen admissible class, $\tau\text{-P}_{\text{adm}}$ is best read as a *property of computations* (specifically, those τ -Tower Machine computations whose internal-depth-polynomial verifier-search relation holds) rather than as a $\text{TIME}(f(n))$ -class in the orthodox sense [14]. The internal collapse $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$ holds in this property-of-computations sense; converting it to a $\text{TIME}(f(n))$ statement requires the export bridge discussed in §14.

The comparison with Wolfram is therefore precise. Wolfram fixes finite machine classes and studies the functions and runtimes that appear within them. The τ spine fixes an internal computation grammar and asks whether a verifier stabilizes through a finite quotient. Both approaches turn an infinite question into finite surfaces one can inspect. But the surfaces are different: finite enumeration in one case, finite interface width in the other.

This difference suggests a productive diagnostic. If a Wolfram-style finite example looks hard because no smaller machine computes it in the chosen class, one should not immediately ask whether it is τ -admissible. One should first ask a weaker question: what is the analogue of its interface? What information must any verifier read to certify the output? Does the difficulty come from a large search space, from an unstable representation, from halting sensitivity, or from the absence of a finite factorization? These questions are not solved in this note, but they turn the bridge into a usable research protocol.

5. A ONE-MINUTE TOY EXAMPLE: ASSEMBLED WITNESS, NOT SCAN

The smallest useful example is deliberately not an NP-complete problem, not a SAT reduction, and not an orthodox complexity reduction. It only illustrates the τ mechanism: when local finite coordinates and a reconstruction theorem are present, witness existence can become witness assembly. Work in the finite quotient

$$Q = \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/5\mathbb{Z}.$$

Suppose the local witness constraints are

$$\begin{aligned} x &\equiv 1 \pmod{2}, \\ x &\equiv 2 \pmod{3}, \\ x &\equiv 4 \pmod{5}. \end{aligned}$$

A flat scan would test residues $0, \dots, 29$ until one satisfies all three checks. CRT instead constructs the address. The three standard reassembly terms are 15, 10, and 6: each is 1 on its own prime component and 0 on the other two after the appropriate local normalization. Thus

$$x \equiv 1 \cdot 15 + 2 \cdot 10 + 4 \cdot 6 = 59 \equiv 29 \pmod{30}.$$

The result can be checked immediately:

$$\begin{aligned} 29 &\equiv 1 \pmod{2}, \\ 29 &\equiv 2 \pmod{3}, \\ 29 &\equiv 4 \pmod{5}. \end{aligned}$$

This toy example captures only the address-construction grammar. It does not prove anything about arbitrary SAT instances, ordinary NP-complete problems, or the Clay P-vs-NP problem. Its purpose is narrower and more useful: it shows the difference between scanning a flat finite space and constructing a global witness from typed local components. In the τ -admissible case, the source-spine theorem claims that the real witness spaces have an analogous finite quotient, local constraint, and canonical reassembly structure. When that structure is absent, the example does not apply. The work was moved from scanning candidate witnesses to constructing the coordinate system in which the witness is already locally determined.

Categorical rendering: CRT reassembly as a pullback square [τ -Effective]. The toy reassembly above is, in the categorical reading, an instance of the *arithmetic fracture square* for $\mathbb{Z}/30\mathbb{Z}$ in the category of finite commutative rings. The Chinese Remainder Theorem states that

$$\mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/2\mathbb{Z} \times_{\mathbb{Z}/1\mathbb{Z}} \mathbb{Z}/3\mathbb{Z} \times_{\mathbb{Z}/1\mathbb{Z}} \mathbb{Z}/5\mathbb{Z},$$

exhibiting $\mathbb{Z}/30\mathbb{Z}$ as a pullback over the trivial fibre (the moduli are pairwise coprime, so all pairwise intersections are trivial; the limit reduces to a product). The local witness constraints ($x \equiv 1 \pmod{2}$, $x \equiv 2 \pmod{3}$, $x \equiv 4 \pmod{5}$) are an element of the product $\{1\} \times \{2\} \times \{4\}$ in the right-hand side; the inverse of the CRT isomorphism delivers the unique element $x \equiv 29 \pmod{30}$ in the left-hand side. The hand computation with reassembly terms $\{15, 10, 6\}$ is the explicit construction of this inverse.

This is the categorical content the source-spine theorem generalises. Under τ -admissibility, the per-prime witness components are objects of the per-prime fibres of the verifier-witness fibration; their compatibility (pairwise coprime moduli; no hidden global consistency constraint) is the fracture-square condition; and the “Product-Meet Collapse” invoked by Theorem 7.1 below is the assertion that the verifier-witness fibration preserves this pullback. In Hopkins–Bousfield language [5], the τ -admissible fragment is the subcategory where the verifier-witness fibration sits over a fracture-square decomposition of the witness quotient. “Witness search as address construction” is then the explicit inverse-image computation along this fibration. The present note does not assert that arbitrary NP witness spaces admit such a fracture-square structure (that would amount to a P-vs-NP claim); it asserts only that, where the fracture structure is present and τ -admissibility holds, search reduces to the inverse-image computation. The logical category of the Product-Meet Collapse generalisation is therefore: *a pullback-preservation property of the verifier-witness*

fibration over the τ -admissible fragment, not a higher-order arithmetic statement and not a sheaf-gluing axiom.

The failure variant is just as important. If the local constraints are not independent, if the moduli are not compatible, or if a hidden global consistency condition remains after local checking, the CRT assembly pattern fails. Such a problem may remain a search problem and is not τ -admissible by this example alone.

6. WITNESS SEARCH AS ADDRESS RESOLUTION

In the orthodox language of NP, a witness is a certificate. It may be easy to check once supplied, even if it is hard to find. The usual anxiety is that the witness space is too large: exponentially many candidate bitstrings, no general deterministic method for locating the right one, and no known proof that such a method cannot exist.

The τ source spine retypes the witness. A witness for an admissible problem is a τ -address at some primorial depth. That address carries coordinates. Its components can be read through the CRT decomposition of $\mathbb{Z}/\text{Prim}(k)\mathbb{Z}$ into per-prime residues. Under τ -admissibility, the verifier factors through finite width, so witness constraints become per-prime constraints that can be searched locally and reassembled globally. The witness is no longer a flat object hidden in a featureless space. It is a structured address whose local components have to cohere.

This is the technical heart of the title. Search becomes structure not because nondeterminism is wished away, but because the object searched for has changed type. In a flat bitstring space, global satisfaction is a meet over constraints with no guaranteed product architecture. In the primorial tower, when the admissibility hypotheses hold, the CRT turns global satisfaction into compatible local construction. The per-prime witness spaces remain finite, and their reassembly is unique.

Relation to the constructive proof-theory lineage..

The reading “witness search as address construction” has a forty-year-old proof-theoretic lineage in constructive logic. The Brouwer–Heyting–Kolmogorov interpretation reads an existence claim $\exists x. \varphi(x)$ as a construction of a witness x together with a verification of $\varphi(x)$; Gentzen’s cut-elimination converts indirect proofs into direct constructions; Martin-Löf type theory [15] encodes witness construction as typed-term inhabitation; Kreisel–Kleene realizability and its descendants [23] make explicit the “witness as constructive object” reading; and modern proof-complexity work synthesises these lineages with computational complexity [14]. The τ -admissibility move asserted here — under finite interface width, witness search admits an address-construction alternative to blind enumeration — sits in this constructive lineage as a typed, modulus-indexed version of the proof-theoretic “existence-with-witness” reading. The novelty of the present note is not the reading itself but the specific finiteness crite-

rion (interface width on a primorial tower) under which it applies; the lineage gives the reading its standing rather than its content.

It is useful to spell this out as a four-step grammar. First, the verifier is typed: it is not an arbitrary external checker but an E_2 -level computation whose observable behavior has a finite interface. Second, the candidate witness is typed: it is represented as an address at a finite primorial depth rather than as an undifferentiated string. Third, the constraints are localized: acceptance conditions factor into conditions on finite per-prime components. Fourth, the local solutions are reassembled by CRT into the global address. At no point is the witness found by a mysterious act of guessing. The work is shifted from blind global enumeration to structured local resolution.

This is also why the address language is not merely metaphorical. In an ordinary NP problem, a certificate can be encoded as bits, but the bit encoding does not by itself explain why the right certificate should be found efficiently. It only gives a representation in which verification is possible. In the τ -admissible case, the representation is tied to a construction theorem: the finite quotient, per-prime decomposition, and unique reassembly are part of the computational grammar. The address is not a label attached after the fact. It is the shape through which the witness becomes constructible.

Proposition 6.1 (Bridge reading of witness search [τ -Effective]). *Within the τ -admissible fragment, witness search is not modeled as an unstructured enumeration of certificates. It is modeled as address resolution: identify the finite quotient, solve the per-prime witness conditions, and reassemble the canonical address by CRT.*

Source-spine proof sketch. The source spine supplies three ingredients. First, finite interface width forces the verifier to factor through a finite primorial quotient. Second, the witness space at that quotient decomposes into per-prime components by CRT. Third, unique CRT reassembly turns compatible local witnesses into a global witness. The cost statement in the source is polynomial in the primorial depth k , with the important caveat that this is not automatically the same as ordinary bit-length polynomiality. \square

The contrast with Wolfram’s nondeterministic machines is now clear. Wolfram’s multiway graphs show that branch availability can shorten paths. The τ source spine asks for something stronger: a deterministic structural procedure that finds and reassembles the witness because the witness space has finite address architecture. A short branch and a constructed address are related ideas, but they are not the same.

6.1 What the toy example did and did not show

The CRT example above is intentionally small enough to be checked by hand. It shows the contrast between a flat search space and an address space whose local components can be

assembled. It does not show that every certificate space has such a product structure. It also does not show that ordinary input length and primordial depth are interchangeable. Those are precisely the questions that any classical export would have to answer.

The didactic point is therefore conditional. If a witness problem has a finite quotient, local constraints, and canonical reassembly, then the right search grammar is construction. If the witness remains a flat certificate with no finite address decomposition, the example has no force. This is why the definitions of τ -admissibility come before the formal equality in this note.

7. THE COMPUTATIONAL BI-SQUARE AND ADMISSIBILITY COLLAPSE

The computational bi-square is the diagrammatic center of the internal claim. Earlier bi-squares in the framework have the same repeated shape: one square records tower coherence, another records spectral naturality, and the pasted rectangle expresses the relevant constraint at that layer. At the computation layer the objects have changed. They are verifier computations and witness spaces. The left square is the execution square; the right square is the witness square. Pasting them yields the Product-Meet Collapse.

In plain language, the Product-Meet Collapse says that, inside the admissible setting, satisfying all witness constraints is realized by constructing the product of compatible per-prime witness components. The meet of constraints becomes the product of components. This is the moment where search becomes structure in the strongest internal sense.

The diagram matters because it prevents the collapse from becoming a black-box slogan. The left side records that computation is still an execution process: an agent acts, a verifier reads a finite interface, and the observable transition must remain coherent through the tower. The right side records that the witness is not merely asserted: its local components have to be natural with respect to the spectral/prime-indexed decomposition. The pasted diagram says that these two requirements are not independent decorations. They meet in the same finite construction surface.

This is also where the τ source spine differs from a lookup-table shortcut. In a finite universe, one can sometimes compute a function by encoding enough answers in a larger machine. That is a representation trick unless it comes with a stable construction rule. Product-Meet Collapse is not a claim that every finite instance can be stored. It is a claim that, for admissible computations, the witness space itself has a decomposition that the solver can use uniformly. The distinction is essential for any future export discussion.

Theorem 7.1 (Framework source-spine theorem: τ -admissibility collapse [τ -Effective]). Scope (load-bearing). *The following equality is internal to the τ -admissible fragment; $\tau\text{-P}_{\text{adm}}$ and $\tau\text{-NP}_{\text{adm}}$ are not the classical Clay classes P and NP*

over unrestricted Turing-machine / ZFC encodings; no claim is made about the orthodox P-vs-NP question.

For τ -admissible witness problems, the internal source spine states

$$\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}} \quad (\text{internal to the } \tau\text{-admissible fragment}).$$

If a verifier has finite interface width, witness search runs in time polynomial in the primordial depth used by the admissible computation; the relationship between primordial depth k and orthodox bit-length n is itself part of the bridge work and is not asserted here as a $\text{TIME}(f(n))$ -class identification (see §3 discussion below).

Source-spine proof sketch. The source-spine proof assembles three claims. Constant-width verification supplies the execution side. CRT witness decomposition supplies the witness side. Product-Meet Collapse pastes the two squares: independently resolved per-prime witnesses assemble into a global witness. Thus verification and search both become polynomial in the internal depth parameter for the admissible fragment. \square

The theorem remains source-spine material [8]. The surrounding public construction grammar can be inspected through the Construction Spine, TauLib, WP003, the Release Manifest, and the Corpus Registry [16, 19, 21, 18, 17].

MAIN NON-CLAIM

The displayed equality is $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$, not an orthodox $P = NP$ theorem. It belongs to τ -admissible computation. This is an internal equality for τ -admissible witness problems. It does not settle orthodox P vs NP over unrestricted Turing-machine / ZFC encodings.

8. WHAT FAILS τ -ADMISSIBILITY?

The collapse claim is credible only if the failure modes are visible. A problem does not become τ -admissible because it has been renamed in τ -vocabulary. It has to earn the finite construction surface. The most important failure modes are:

These are not embarrassment clauses; they are part of the theorem's meaning. A serious future benchmark should contain negative rows where the admissibility test fails. Wolfram-style finite microcosms are useful here because they can produce exactly the kind of stubborn runtime outliers and branch structures that test whether a proposed τ encoding is real structure or only a renamed search. This is an internal equality for τ -admissible witness problems. It does not settle orthodox P vs NP over unrestricted Turing-machine / ZFC encodings.

9. FINITE EXPLORATION VS FINITE INTERFACE WIDTH

We can now state the first major comparison. Wolfram uses finite enumeration. The τ spine uses finite interface width. Both are ways of refusing to let the infinite formulation do all the work.

Finite ruliological exploration

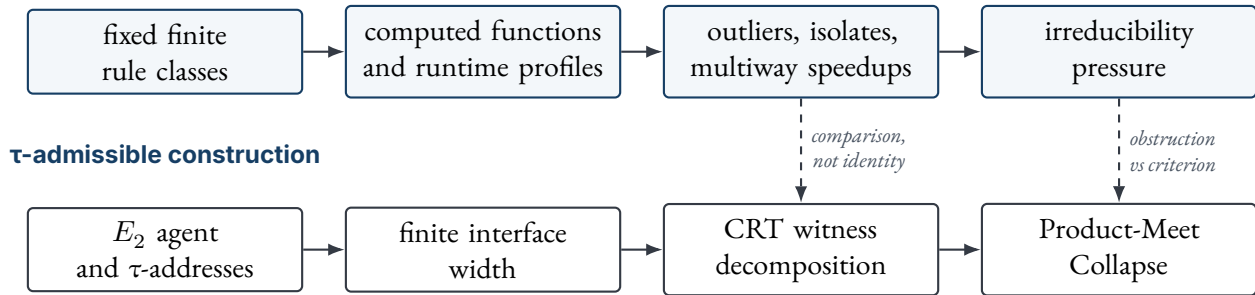


Figure 1. Search becomes structure only on the lower path. Wolfram’s finite microcosms make hardness surfaces visible from below. The τ path turns search into construction only after finite interface width and CRT witness decomposition are available.

Table 4. Failure modes for τ -admissibility. The restrictive nature of this table is what prevents $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$ from being an unguarded $\text{P} = \text{NP}$ claim.

Failure mode	Why τ -admissibility fails	Consequence
Unbounded witness representation	The witness cannot be finitely address-coded at the cutoff.	The problem remains search.
Hidden global consistency No reconstruction theorem	Local checks do not reconstruct the global witness. No CRT-, Yoneda-, or Hartogs-style assembly route is available.	Additional global search remains. Existence has not become construction.
Unbounded interface width External semantic oracle	Local data require growing cross-talk as instances vary. Verification depends on meaning outside formal address data.	The verifier is not boundedly admissible. Runtime or semantics have been hidden.
Encoding-dependent shortcut	The collapse depends on changing the problem representation illegitimately.	The result is not exportable.

Finite enumeration starts by fixing a rule class and asking what appears inside it. One can count machines, compare functions, identify isolated representatives, and inspect runtime profiles. The gain is empirical specificity. The cost is class relativity. A fact about $s = 3, k = 2$ machines may change when $s = 4, k = 2$ machines are admitted.

Finite interface width starts differently. It asks whether a verifier stabilizes through a finite quotient independent of deeper tower levels. The gain is theorem-grade internal structure. The cost is admissibility: only computations satisfying the finite-width criterion fall inside the collapse. A computation that truly requires unbounded depth remains outside until additional structure is supplied.

This is why the note’s title should not be read as a universal claim that every search becomes structure. The statement is conditional. Search becomes structure when a finite address architecture exists, when the verifier stabilizes at finite interface width, and when the witness space factorizes in a way compatible with construction. Wolfram’s finite microcosms help us see why those conditions matter: without them, runtime behavior can remain wild, class-relative, and irreducibly difficult to summarize.

9.1 Two finite moves, two different risks

The two finite moves have opposite failure modes. Finite enumeration can be too weak because it remains tied to the chosen class. A finite machine microcosm may contain a genuine

outlier, but another class may remove it. The result is empirical richness without immediate generality. Finite interface width can be too strong because it excludes computations whose observable behavior does not stabilize. The result is theorem-grade structure, but only inside a typed fragment.

The future research program has to keep both risks visible. If the τ side ignores Wolfram-style finite exploration, it risks becoming too formal: internally elegant but insufficiently tested against the messy behavior of small rule universes. If the Wolfram side ignores finite interface conditions, it risks staying descriptive: rich catalogs of difficulty without a criterion for when a witness space has become constructible. The bridge note is valuable precisely because it can let each side expose the other’s blind spot.

This is also why the comparison should be carried out with examples, not only with vocabulary. A finite rule example can be annotated by the questions it raises for τ : what is the interface, what is the witness object, what would count as a finite quotient, and what would fail if no such quotient exists? Conversely, a toy τ -admissible example can be translated into a Wolfram-style finite exploration: what does the deterministic search look like before the address decomposition is made visible? The two directions would not prove equivalence. They would make the bridge measurable.

10. NONDETERMINISM VERSUS WITNESS CONSTRUCTION

Table 5. Core comparison axes. The table is intentionally asymmetric: Wolfram supplies finite microcosms of difficulty; the τ spine supplies a conditional construction theorem under admissibility.

Axis	Wolfram ruliology	τ computation spine	Bridge sentence
Finite structure	Enumerated finite machine universes.	Finite interface width and finite quotient factorization.	Both make finite readout explicit.
Search	Runtime outliers and class-relative difficulty.	Witness search becomes address resolution under τ -admissibility.	Wolfram studies hard search; τ specifies when search becomes construction.
Nondeterminism	Multiway branching can shorten paths.	Witnesses are canonical addresses.	Reach and construction must be kept distinct.
Irreducibility	Computational irreducibility obstructs shortcutting.	Inadmissibility marks failure of finite collapse.	Irreducibility is pressure; admissibility is criterion.
Infinity	Ruliad/everything-machine gestures toward maximal rule-space reach.	τ -native computation uses a particular primordial construction grammar.	Philosophically adjacent, architecturally different.
Orthodox export	Essay does not settle classical P vs NP.	τ collapse is internal unless a separate bridge is supplied.	Shared humility makes the bridge note viable.

The second comparison is the most delicate. In Wolfram's essay, nondeterminism is implemented by allowing a choice among rules at each step. The resulting multiway graph contains many possible paths. Some paths compute outputs faster than the deterministic rule path. In the limiting everything-machine picture, nondeterminism becomes maximal reach: from an input, many outputs become accessible by structurally minimal paths.

In orthodox NP language, nondeterminism is often described by saying that the right witness can be guessed. That phrase is useful pedagogically, but it hides the key issue. A short accepting path may exist without a deterministic polynomial method for finding it. A multiway graph may contain the route, but the route may not be visible to a bounded solver. The real question is not merely whether the path exists. It is what kind of structure makes the path constructible.

The τ answer is not "branch harder." It is "change the type of the witness." Under admissibility, the witness has address structure. The search space is not just a cloud of possibilities; it is a finite quotient with per-prime components. The deterministic construction procedure is not a blind traversal of branches. It is a decomposition and reassembly procedure.

Remark 10.1 (Reach is weaker than construction). A nondeterministic reachability statement says that some path exists in a multiway graph. A τ -admissible construction statement says that the witness can be recovered from finite address components and reassembled. The second is stronger and more typed than the first.

This distinction also protects the note from an attractive but false symmetry. It would be easy to say that Wolfram's nondeterministic speedups are the same thing as τ witness construction. They are not. Wolfram's examples show how branch availability can reduce observed path length in finite rule universes. The τ spine gives an internal criterion for when a witness space ceases to be featureless and becomes

constructible. The bridge is that both focus on the structure of possible paths. The boundary is that only the τ side claims an internal construction theorem, and only under admissibility.

The distinction has a practical consequence for future benchmarks. A candidate example should not be counted as a τ analogue merely because nondeterminism speeds it up. The first diagnostic should be: does the speedup identify a reusable witness structure, or only a fortunate branch? If the shortest branch depends on global search through the multiway graph, then the example remains a reachability example. If the branch can be replaced by a local decomposition-and-reassembly procedure, then it starts to resemble the τ witness grammar.

This is a useful discipline because many popular accounts of P vs NP slide too quickly between "the answer can be checked" and "the answer can be found." Wolfram's finite graphs make the slide visible: the path can be in the graph without being efficiently discoverable. The τ source spine turns that visibility into a criterion: discoverability comes from finite address structure, not from the mere existence of a branch.

11. IRREDUCIBILITY, INADMISSIBILITY, AND INFINITY

Wolfram's essay repeatedly returns to computational irreducibility: the possibility that no shortcut captures the behavior of a computation without effectively running it. In the P-vs-NP context, this pressure appears as runtime wildness, halting uncertainty, difficult limit behavior, and the suspicion that exceptions may not average away. The essay suggests that computational irreducibility and undecidability pressure are not rare corner cases but pervasive features of rule space.

The τ source spine uses a different vocabulary. It does not begin by asserting irreducibility. It defines admissibility. A computation is admissible when finite interface width exists. A computation is inadmissible, for the purposes of this note, when no such finite stabilization is available. The relationship is suggestive but not identity. Irreducibility is a pressure against

shortcutting. Inadmissibility is a failure to meet a particular internal finite-width criterion.

Runtime irreducibility and descriptive irreducibility..

Wolfram's computational irreducibility is itself two-sided. *Runtime irreducibility* says that no shorter computation reproduces the behaviour in fewer steps: simulation requires the simulator to run essentially as long as the simulated process. *Descriptive irreducibility* says that no shorter description of the behaviour exists at all: even a perfect oracle for the long-run state cannot be replaced by a more compact characterisation. The τ -admissibility criterion engages the runtime side: where finite interface width holds, the verifier-search relation collapses, and runtime irreducibility is locally defeated by address construction. The τ -admissibility criterion *does not* engage descriptive irreducibility: even an admissible computation may still admit no closed-form characterisation shorter than its address-resolution procedure, and the present note makes no claim that the constructive reading defeats descriptive irreducibility. The Wolfram essay's strongest claims sit on the descriptive side; the τ -admissibility collapse operates on the runtime side. The two sides do not contradict each other.

This contrast helps both sides. Wolfram helps us resist a too-easy reading of the τ collapse: if finite-width structure is absent, wildness and non-shortcut behavior should be expected, not treated as a minor technicality. The τ spine helps us sharpen the positive side: when admissibility is present, the reason search becomes constructive is not that all computations are reducible in general, but that this computation has finite quotient structure.

The infinity comparison is similar. Wolfram's everything-machine and ruliad language points toward maximal computational reach: all rules, all paths, all computations, with bounded observers sampling slices. The τ spine points toward a different treatment: a primordial tower whose finite stages, potential limits, and address structure delimit what a τ -agent can inspect and construct. Both frameworks make observer boundedness visible. But maximal reach and admissible construction are not interchangeable.

This difference gives the note its negative testing surface. If a candidate problem continues to require ever deeper primordial coordinates, or if the verifier cannot be shown to factor through a stable finite quotient, then the τ collapse should not be invoked. Such a case may be computationally irreducible in a Wolframian sense, inadmissible in the τ sense, both, or neither. The categories must not be collapsed without a bridge argument.

The most interesting future cases may be mixed. A finite rule universe may show irreducible-looking behavior that later turns out to have a hidden address decomposition under a different representation. Conversely, a τ -like encoding may appear promising until one discovers that the required interface width grows with the input. The honest research program is to sort these cases, not to declare them solved in advance.

That is why inadmissibility is not a failure of the framework; it is one of its necessary boundary markers.

11.1 Secondary echoes

Book II supplies a finite/readout substrate echo: finite-stage data, ultrametric depth, profinite limits, and stagewise finite linear algebra [9]. Book VI supplies application echoes: evolution and cognition as processes that navigate large search spaces through local exploration, selection, caching, and approximation [11]. Book VII supplies the readout echo: apparent randomness and probability arise when observers cannot resolve all internal structure [10]. These echoes are useful for later prose, but they are not the proof spine of this note. They explain why the search/structure pattern recurs; they do not prove a complexity-theoretic claim.

These secondary echoes are contextual only; they are not part of the proof spine of this note. They can show that the same readout pattern recurs across the framework: finite carriers before limit claims, local navigation before global comprehension, observer resolution before probabilistic summary. But they should not be allowed to blur the proof spine. The P-vs-NP comparison lives or dies on the computation/admissibility material, not on a general family resemblance across the books.

12. THE RULIAD IS NOT CATEGORY τ

Because the two projects are philosophically close, the distinction must be made explicitly. Wolfram's ruliad is a maximal object: the entangled limit of all possible computations. It is not a single machine, not a single model, and not a particular address grammar. It is the all-paths object from which bounded observers sample slices.

Category τ , by contrast, is a specific construction framework. It has generators, axioms, a primordial tower, CRT decomposition, enrichment levels, τ -addresses, and proof-organizing diagrams such as the computational bi-square. Its internal P-vs-NP claim is not that all paths exist. It is that, under admissibility, the witness space has enough finite address structure for search to become construction.

The productive comparison is therefore this:

The ruliad foregrounds maximal computational possibility and observer sampling. The τ computation spine foregrounds admissible construction and finite readout. Both make the hidden architecture of computation visible, but they do so with different primitives and different proof obligations.

This distinction should stay visible in any public version of the note. The temptation will be to say that the ruliad and Category τ are two names for the same intuition. They are not. They are neighboring frameworks that converge on a shared dissatisfaction with black-box complexity abstractions. That convergence is enough. It is more honest, and more interesting, than forced equivalence.

One way to state the distinction is by asking what each framework treats as primitive. The ruliad begins from the space of possible computations and asks how observers are embedded in that enormous object. Category τ begins from a constructed address calculus and asks how layers of structure, readout, computation, and observer access are built. The ruliad is maximal before observation; Category τ is constructive before maximality. This is not a ranking. It is a difference in explanatory direction.

Equivalently: the ruliad is a possible-computation space, while Category τ is a constrained constructive architecture. That difference is exactly why the comparison is worth making. A possible-computation space asks what exists in the maximal rule ensemble. A constructive architecture asks what can be built, read, checked, and reassembled under the finite constraints of the framework.

That difference matters for P vs NP. A maximal computational ensemble can make the landscape of possible paths vivid, but it does not by itself select a deterministic construction procedure. A constructed address grammar can select a construction procedure, but only after its admissibility hypotheses have been checked. The two perspectives are therefore complementary in the ordinary sense: each can expose questions the other tends to leave implicit. They are not complementary in the stronger sense of forming two halves of a single proof.

What the ruliological programme positively contains, beyond the engagement of this note.. For completeness, the ruliological programme contains substantial positive content that this bridge note does not engage and does not claim to complete. The Wolfram Physics Project [25, 26] has developed (i) a hypergraph-rewriting account of spacetime emergence and of relativistic / general-relativistic behaviour from rule systems [12]; (ii) an observer-theoretic account of why bounded observers perceive lawful coarse-grained physics from inside the ruliad; (iii) a multiway / multicomputation account of quantum-mechanical phenomena via branchial-graph geometry [2]; and (iv) the broader ruliological methodology of exhaustive enumeration as scientific practice [24]. None of these load-bearing pieces of the ruliological programme is engaged in the present note. The note is a narrowly scoped bridge — it imports the ruliological essay as a finite-systems comparator for a separate τ -admissibility claim — and is emphatically not an outside-framework summary or refutation of any of the ruliological positive content. A senior ruliologist reading this note should treat it as outside engagement on one bounded comparator question, not as response to the Wolfram Physics Project as a whole.

13. PUBLIC SOURCE AND CONSTRUCTION CONTEXT

The τ side is grounded in the computation/admissibility source spine cited as [8], and the public Panta Rhei site exposes the surrounding construction grammar through inspectable routes. The substantive 100-step construction spine itself — the typed

$E_0 \rightarrow E_1 \rightarrow E_2 \rightarrow E_3$ build from finite τ -kernel to reflective ontic closure — is given as the PANTA RHEI CONSTRUCTION SPINE whitepaper (OSF 10.17605/OSF.I0/6GJ9K, CC-BY 4.0, RC v3.0) [7]; that whitepaper is the recommended companion read for any inspection of the categorical construction this note rests on. Surrounding public routes provide auxiliary inspection: the Construction Spine site route, TauLib Corpus, TauLib verification route, WP003 TauLib Technical Overview, Release Manifest, and Corpus Registry [16, 19, 20, 21, 18, 17]. The internal source spine [8] remains the technical reference; the construction-spine whitepaper [7] gives the substantive 100-step public-facing build, and the surrounding public routes give the contextual surfaces for what the framework means by construction, registry, formal route, and release discipline.

Public inspection surfaces. The construction-spine whitepaper [7] is the primary public-facing artifact: a 100-step exhaustive build with explicit claim boundary, status-tag discipline ([formal] / [hinge] / [bridge] / [readout] / [evs]), and an audience-calibrated 45–60 minute overview / 90–120 minute full inspection time budget. The Construction Spine site route gives a public route into the staged build order. The TauLib Corpus and Verify/TauLib pages expose the formalization and inspection surface. WP003 records the TauLib technical overview and trust budget (Lean library infrastructure, complementary to the construction-spine whitepaper which gives the substantive build). The Release Manifest pins the public verification state. The Corpus Registry gives the object-indexed route into definitions, theorems, structures, and dependencies. These surfaces are contextual inspection routes, not substitutes for the internal source-spine theorem.

The Wolfram-side context is similarly layered. The primary external anchor for this note is the January 2026 essay itself [27]. The Wolfram Institute ruliology route is used only as contextual support for the word “ruliology” and the broader programmatic setting [28]. It is not used as a second proof source for the P-vs-NP argument.

This public-context split matters for outreach. The note can be sent to a Wolfram/Wolfram Institute reader without asking them to accept private terminology on trust: the public pages show the construction spine and verification surfaces; the manuscript explains the guarded comparison; and the claim boundary remains explicit at each step.

14. ORTHODOX P-VS-NP GUARDRAILS

The most important section of this note may be the one that says what it does not do. The classical P-vs-NP problem is a problem in orthodox complexity theory, formulated over Turing machines, polynomial time, decision languages, encodings, reductions, and proof systems such as ZFC [6]. A framework-internal theorem about τ -admissible computation does not automatically export to that setting. A finite ruliological experiment does not settle an asymptotic theorem. A

philosophical comparison between rule-space exploration and address construction does not replace a proof.

The τ source spine is itself explicit about this boundary. It distinguishes the internal admissibility collapse from the classical Turing-model question. It treats the bridge to ZFC as a separate issue, and parts of the provability/independence discussion are horizon-limited or conjectural. This note should preserve that humility because it is not a weakness. It is the condition under which the comparison becomes credible.

Position of the τ -admissibility collapse relative to ZFC.. For precision, the present note adopts the following position. The τ -admissibility collapse $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$ is asserted as *internally provable in the source-spine framework of [8]*; that framework is in turn *built inside* ZFC but its computation/admissibility layer sits at a stratum (E_2 , self-referential typed codes) that is one level above the orthodox Turing-machine/string-encoding layer at which the Clay problem is posed. The *export bridge* — a translation functor or definitional interpretation that would carry the internal collapse to an orthodox $P = NP$ or $P \neq NP$ statement over unrestricted Turing machines — is itself *conjectural with respect to ZFC*: this note does not assert that such a bridge exists, that it is consistent, or that it is independent of ZFC. We do not claim $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$ is itself a Σ -statement at any specified arithmetic complexity; we do not claim the τ -admissibility predicate Adm_τ is definable in ZFC at any specified level; we do not assert that any orthodox separation result ($P \neq NP$) would follow from, or be precluded by, the internal collapse. Among the four standard readings of “ZFC provability horizon” — (a) the collapse is ZFC-provable, (b) the framework is ZFC-built but at a different computation-layer stratum, (c) the bridge is ZFC-independent, (d) the bridge is conjectural w.r.t. ZFC — this note adopts (b) for the internal collapse and (d) for the export bridge.

Position relative to known barriers.. The well-known meta-mathematical barriers for orthodox P-vs-NP — the relativization barrier of Baker, Gill, and Solovay [4], the natural-proofs barrier of Razborov and Rudich [22], and the algebrization barrier of Aaronson and Wigderson [1] — do not bite on the present bounded claim. These barriers obstruct proving an orthodox separation $P \neq NP$, which the present note explicitly does not assert. The bounded internal claim $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$ *within* the τ -admissible fragment is asymmetric in the opposite direction: it is a collapse-inside-a-fragment, not a separation across complexity classes. A reader concerned that the τ -framework’s positive bridge work might silently entail a separation that triggers Razborov–Rudich naturalness, algebrization, or relativization should note: the bridge construction is *conjectural w.r.t. ZFC* (per the previous paragraph) and the export to orthodox complexity classes is not asserted. The barriers therefore do not apply to what is here, and the framework’s claim is correspondingly weaker than its language sometimes suggests.

There is a second guardrail, subtler but just as important. The note should not hide behind internal vocabulary. If it uses $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$, it must say in ordinary prose what the subscript does. It marks the admissible fragment. It marks the finite-interface-width condition. It marks a computational grammar different from the orthodox Turing tape. The subscript is not decoration. It is the boundary.

14.1 The three claim tiers

The note keeps three tiers separate. The first tier is *τ -internal*: within the typed admissible fragment, the source spine states $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$. The second tier is *τ -native/physical*: the framework may later interpret computation inside physical or observer-relative readout settings, but those claims inherit additional assumptions. The third tier is *orthodox export*: a statement about standard Turing-machine languages, polynomial time in ordinary input length, reductions, and proof in a background system such as ZFC.

The present note uses the first tier, mentions the second only as future context, and refuses the third as a current claim. This is not rhetorical timidity. It is source fidelity. The internal theorem and the external Clay problem are not the same proposition with different fonts. They have different machines, different encodings, different size measures, and different proof obligations.

The safest public wording is therefore not “ τ solves P vs NP.” It is: inside τ -admissible computation, witness search is retyped as address construction, yielding $\tau\text{-P}_{\text{adm}} = \tau\text{-NP}_{\text{adm}}$; the export to the orthodox P versus NP problem remains a separate bridge problem. This is an internal equality for τ -admissible witness problems. It does not settle orthodox P vs NP over unrestricted Turing-machine / ZFC encodings. That sentence is less dramatic. It is also the sentence that can survive scrutiny.

15. FUTURE BRIDGE TESTS

The best ending for this bridge note is not triumph but work. The bridge between finite ruliology and τ -admissible construction suggests concrete next tests.

Finite benchmark ledger. The first test is documentary: reproduce a small Wolfram-style finite machine ledger and annotate it with the τ vocabulary. Which examples look like pure runtime outliers? Which look like branch-reach examples? Which have any plausible finite-interface-width analogue? This would not prove anything about orthodox complexity, but it would make the comparison less literary and more inspectable.

The ledger should record at least five fields: machine class, computed function or finite input-output map, runtime profile, nondeterministic shortening behavior if present, and the proposed τ diagnostic. The τ diagnostic should be allowed to say “none.” That negative outcome would be scientifically useful, because it would prevent the comparison from becoming

Table 6. Guardrail ledger. The same phrase can be safe or unsafe depending on the tier in which it is used.

Tier	Safe statement	Risk	Required discipline
τ -internal	τ -P _{adm} = τ -NP _{adm} for τ -admissible computation.	Medium: readers may drop the subscript.	Keep “ τ -admissible” and “internal” near every strong claim.
τ -native / physical	Physical or τ -native extensions are framework claims under stated admissibility assumptions.	High: sounds like a headline about all computation.	Use only as later extension, not as the note’s main public claim.
ZFC / Turing / Clay	The orthodox bridge is not established by this note.	Critical: readers infer a Millennium-problem claim.	State the non-claim in introduction, theorem discussion, and conclusion.
Wolfram anchor	The essay is a finite ruliological comparator.	Medium: anchor may be misread as validation.	Call it an anchor or comparator, not evidence for τ .

an all-purpose translation machine.

Witness-decomposition examples. The second test is constructive: build toy witness spaces at small primordial depth and show explicitly how CRT decomposition differs from branch enumeration. A good example would display a flat candidate search, the per-prime decomposition, and the unique reassembly step side by side. The point would be to let readers see why “reach” and “construction” are different claims.

Subsequent benchmark examples should include the actual residues and the actual reassembly calculation. The purpose is not to impress with formalism. It is to let a skeptical reader check the claim by hand. A good example should fit on one page: problem statement, flat witness space, local residue constraints, CRT reassembly, and the explicit note that the example is τ -internal rather than a classical reduction.

Irreducibility boundary map. The third test is negative: identify examples where the finite-width criterion fails or remains unknown. This is essential. A bridge note that only shows positive collapse risks sounding like a universal shortcut story. A serious research note should say where search does not yet become structure.

The boundary map should distinguish at least three cases: known admissible, known inadmissible, and unknown. It should also distinguish “not yet encoded” from “encoded but width appears to grow.” Those are different failures. The first is a missing translation; the second is a structural obstruction. Keeping them separate would make the note a better research tool.

Orthodox export checklist. The final test is a checklist for any future classical export. Such an export would need to specify the encoding, the input-size metric, the relation between primordial depth and bit length, the verifier model, the reduction notion, and the proof system in which the claim is made. Until that checklist is satisfied, the classical Clay problem remains outside the claim.

This checklist is deliberately demanding. It includes: an encoding from ordinary decision-language instances into τ -addressed objects; a size comparison theorem relating bit length and primordial depth; a proof that classical polynomial-time verifiers translate into finite-width τ -verifiers, or a clear

statement of the restricted class that does; a deterministic construction algorithm whose cost is polynomial in the exported size measure; and a proof environment in which the translation is formalized. If any item is absent, the correct public statement is “internal τ result with open export,” not “classical P equals NP.”

Table 7. Export checklist for any future orthodox P-vs-NP bridge. The present note does not satisfy this checklist; it records what would have to be shown.

Export requirement	Question
Encoding	Can orthodox instances be mapped to τ -address objects without exponential blowup?
Verification preservation	Do τ -local checks correspond to ordinary polynomial-time verification?
Reconstruction cost	Is reassembly polynomial in orthodox input length, not only in primordial depth?
Uniformity	Is the construction uniform, rather than instance-specific advice?
Semantic closure	Are all meanings carried by formal address data, with no hidden semantic oracle?
Witness return	Does the constructed τ -witness translate back to a valid orthodox witness?

Ruliological counter-tests. A useful bridge should also invite counter-tests from the Wolfram side. If finite machine exploration produces examples where nondeterministic speedup appears robust across many expanded classes, but no finite address decomposition can be found, those examples become pressure tests for the τ admissibility criterion. Conversely, if a τ toy example has a clean address decomposition, one should ask whether finite rule enumeration can rediscover that structure or merely observe the speedup. Both outcomes would teach something.

16. CONCLUSION: THE USEFUL CONVERGENCE

The useful convergence is not that Wolfram and the τ -framework say the same thing. They do not. Wolfram's essay explores finite computational microcosms and finds wildness: runtime outliers, isolate machines, nondeterministic speedups, everything-machine reach, and irreducibility pressure. The τ computation spine specifies an internal criterion under which search becomes construction: finite interface width, structured witness addresses, CRT decomposition, and Product-Meet Collapse.

The bridge is methodological. Both approaches reject the idea that computation can be understood only from a fully abstract asymptotic distance. Both insist that rule space, representation, observer access, and infinity matter. Both make visible the hidden architecture under the word "search."

But the difference is just as important. Wolfram shows how hard search can look when finite rule universes are explored from below. The τ -framework claims that search becomes structure only inside a specific admissible address grammar. The note's title is therefore not a universal slogan. It is a conditional research program:

Search becomes structure when the witness space has earned enough finite address architecture for construction to replace blind enumeration.

That is already a strong claim. It is strong enough to motivate a research note, strong enough to compare with Wolfram's ruliological microcosms, and bounded enough to send into public discussion without pretending that the orthodox P-vs-NP problem has been settled. This is an internal equality for τ -admissible witness problems. It does not settle orthodox P vs NP over unrestricted Turing-machine / ZFC encodings.

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We gratefully acknowledge Stephen Wolfram for the open-access deposit of the 2026 essay [27] and for the broader ruliological programme on which the bridge sections of this note rest. The vocabulary of the *ruliad* [26], of *multiway* systems, and of *computational irreducibility* deployed throughout §§2–3 originates in the Wolfram Physics Project [25, 24]. The hypergraph-rewriting and multicomputation-foundations work of Jonathan Gorard [12] and Xerxes Arsiwalla [2], together with the broader Wolfram Institute community (Piskovskiy, Elshatlawy, and others), built the rule-system / multiway / ruliad infrastructure that the present bridge engages from outside; that work is the methodological context for the asymmetry framing of §2. The Hopkins–Bousfield arithmetic fracture-square machinery [5] sharpened the categorical reading of the CRT example in §5; the constructive proof-theory lineage of Troelstra [23] and Martin-Löf [15], and the proof-complexity synthesis of Krajíček [14], sited the "witness search as address construction" reading of §6 within a forty-year-old programme of constructive logic. The realizability home for the τ -admissibility predicate is the effective topos [13], with standard categorical infrastructure per Awodey [3]. The natural-proofs, algebrization, and relativization barriers [22, 1, 4] shaped the bounded-claim discipline of §14. The companion artifact for the substantive 100-step typed construction spine referenced throughout this note is the PANTA RHEI CONSTRUCTION SPINE whitepaper [7] (OSF 10.17605/OSF.I0/6GJ9K), published as a public-facing release candidate concurrently with this note. None of these acknowledgements should be read as implying endorsement of the τ -framework by the cited authors or programmes; the present note's positions are the authors' own. Engagement, criticism, and falsification of any of the bridge-section claims would be welcomed.

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