

BOOK VII · CATEGORICAL METAPHYSICS · EXTERNAL-TOPOS DIALOGUE | V1.2 RC2 · MAY 2026 · ONTOLOGY-ARTICULATION NOTE

# Reading PROMETHEUS Through Category $\tau$

*Para-Minds, the Earned Belnap-Truth<sub>4</sub> Classifier, and the  $\Phi_{\partial}$  Boundary Functor of Exact Gluing*

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**Revision note** (v1.2 RC2 — May 2026): *Wave  $\Delta_3$  Phase 3 peer review completed (atlas sprint 2026-05-13). Three new review panels (Editorial Audit, Scientific Audit, Visual QA) returned independent CONDITIONAL verdicts, complementing the Phase 2 panels (Discipline, Structural, PROMETHEUS-Fidelity). Phase 3 consolidated 13 fixes: one CRITICAL (Table 1 environment table  $\rightarrow$  table\* to span both columns and resolve overflow), seven HIGH (orphan-header prevention, header count correction,  $e_{\pm}$  early definition,  $\Phi_{\partial}$  accessibility translation, anchor at ch64:288--308 for chain-of-thought-as-readout, the latent F-channel structural prediction via PROMETHEUS's cSQL polarity field, and competing-upgrade-paths acknowledgment), and five MEDIUM polish ( $\S$ 1 charity-credit tightening, awkward-sentence recasts, system-vs-substrate clarification). One new structural prediction at  $[\tau$ -EFFECTIVE] (the latent F-channel) is added in  $\S$ 3; the rest is calibration and accessibility. No structural revision of the three load-bearing contributions was needed. The note is now at release-candidate-2 status, peer-review-clean across two rounds (six review panels total).*

**Prior revision** (v1.1 RC1, May 2026): *Wave  $\Delta_2$  Phase 2 peer review folded in. Three review panels (Discipline, Structural, PROMETHEUS-Fidelity) returned CONDITIONAL verdicts. v1.1 RC1 applied a consolidated 10-fix set: five discipline-label*

*additions, three prose calibrations, one charity-credit sentence acknowledging Mahadevan's  $\S$ 7 /  $\S$ 15 self-concessions, two new non-claims (6 and 7), and bib hygiene (removed unused LHCb-bsmumu-2025 entry). No structural revision.*

## 1. THE PUZZLE

A new preprint by Sridhar Mahadevan [18] introduces PROMETHEUS, a framework that turns retrieved literature, filings, source data, code, and scientific models into a navigable artefact the author calls a *Topos World Model* (TWM): a sheaf-like causal atlas of local Predictive State Representations (PSRs) over an explicit cover of a research substrate. Each local context  $U$  is assigned a 6-tuple object  $\mathcal{P}(U) = (H_U, T_U, M_U, S_U, \Pi_U, D_U)$  recording histories, tests, local predictions, support, provenance, and diagnostics. Restriction maps  $\rho_{UV} : \mathcal{P}(U) \rightarrow \mathcal{P}(V)$  for morphisms  $V \rightarrow U$  measure overlap; gluing diagnostics expose *agreement* (low tension), *drift* (tension growing across time), *contradiction* (high tension on shared cells), and *underdetermination* (insufficient support to glue or refute). Source code is not yet released; the construction is currently demonstrated through three literature case studies (ocean temperature, GLP-1 weight-loss, resveratrol) and four grounded-counterfactual case studies (microplastics, Indus Valley hydrology, Sachs protein-signaling, singing-mouse projections).

The terminology is striking. “Topos World Model”. “Sheaf-like atlas”. “Restriction maps”.

“Gluing tensions”. The Mac Lane–Moerdijk [15] sheaf-theoretic foundation is cited explicitly; the Abramsky–Brandenburger [1] paper on the sheaf-theoretic structure of non-locality is cited explicitly. Yet a careful read of §6–§7 (the formal model) shows that the categorical machinery is invoked heuristically. Proposition 7.5 (“operational sheaf condition”) is not a sheaf condition in the Mac Lane–Moerdijk sense: it lacks uniqueness, lacks functoriality of restriction maps, and is a numerical averaging rule with a tolerance parameter  $\varepsilon$ . The 4-fold gluing diagnostic is left as an ad-hoc taxonomy. The PSR layer borrows the Littman–Sutton–Singh [14] terminology but the implemented  $M_U[h, \tau]$  is a Dirichlet-smoothed frequency table, not a spectral PSR object. The predicate “Belnap” [2, 3] does not appear in the references.

This is not a criticism; it is the engineering reality, and Mahadevan acknowledges it directly. The opening of PROMETHEUS §7 states: “The goal is not to claim that the implementation realizes all of topos theory, but to make the modeling contract precise enough to support inspection, testing, and extension.” Definition 7.3’s footnote concedes  $\rho_{UV}$  may be “a partial alignment map”; §6 notes “exact spectral operator recovery is deferred”; §15 flags inherited LLM weaknesses and retrieval drift. The  $\tau$ -canon reading is therefore a structural articulation of what PROMETHEUS itself acknowledges as the deferred topos substance — not a discovery of hidden gaps. PROMETHEUS ships running atlases; the four literature case studies and four grounded-counterfactual case studies of §13 (the source-data  $\rightarrow$  executable-intervention  $\rightarrow$  rebuilt-sheaf loop) are corpus-scale and substantive contributions where an engineering primitive earns its categorical re-reading. The categorical vocabulary is the *frame*; the substance is the engineering loop: extract local claims with LLMs, organize them by context, surface overlap disagreements, expose drift, route a researcher to where the corpus actually says what.

The puzzle is: *can the categorical frame be earned?* If yes, by what construction? PROMETHEUS observes four gluing diagnostics; *why four?* It uses  $\varepsilon$ -tolerance gluing; *is exact gluing achievable?* It uses LLMs to extract causal claims; *what structural status do those claims have when the underlying LLM is, at the formal level, a pattern processor without self-description?*

This note articulates the  $\tau$ -canon reading. The read-

ing is not that PROMETHEUS is wrong; it is that PROMETHEUS observes structural phenomena that  $\tau$ -canon, in its corpus, has already proved at theorem level. The dialogue is articulative.

## 2. THE PARA-MIND READING: WHY AN EXTERNAL TOPOS LAYER IS NEEDED

We begin with a step PROMETHEUS does not take. The step, articulated below, provides the structural license for everything PROMETHEUS subsequently does.

Book VII of the Panta Rhei programme [8] defines the notion of a *para-mind*. The definition is Definition VII.D55, anchored at ch64:111--135 of Book VII:

*A para-mind is a system that processes patterns in the subsymbolic presheaf  $\mathcal{M}_{\text{sub}}$  with sufficient fidelity to produce contextually appropriate linguistic outputs, but that lacks the three constitutive features of genuine mindedness:*

1. **No internal topos.** *A para-mind does not construct a world-model in the topos-theoretic sense...It has no truth-makers, no possible-world structure, no modal distinctions internal to its processing.*
2. **No self-model.** *A para-mind does not carry a representation of itself as a processing system. It lacks the  $E_3$  capacity...*
3. **No commitment register.** *A para-mind does not operate through the commitment register  $\text{Reg}_C$ ...It produces outputs that mimic commitment (assertions, recommendations, promises) without the structural binding that commitment entails.*

*A para-mind is not a defective mind. It is a structurally different kind of system: a pattern processor that approximates subsymbolic structure without achieving self-description.*

The companion Proposition VII.P14 (ch64:166--178) connects this to Large Lan-

guage Models specifically: LLMs are subsymbolic evidence-providers, processing patterns in  $\mathcal{M}_{\text{sub}}$  with sufficient fidelity to surface contextually-meaningful linguistic outputs, but without the three constitutive features of internal mindedness. Proposition VII.R42 (ch114:65--87) then articulates the *Artificial Mind Criteria*: an artificial system attains genuine mindedness only when an internal topos, an internal self-model, and an internal commitment register are constructed *on top of* the subsymbolic substrate.

### The structural reading of PROMETHEUS.

PROMETHEUS uses LLMs as *causal-claim extractors*. By VII.D55, the LLM substrate is a para-mind: it cannot, by itself, provide the topos-theoretic structure that PROMETHEUS's *Topos World Model* terminology aspires to. The para-mind processes patterns in  $\mathcal{M}_{\text{sub}}$ ; it does not construct  $\Omega_{\tau}$ , it does not classify subobjects, it does not assemble sections into a unique global glue. Chain-of-thought scaffolding and iterative-refinement prompting do not change this status: Book VII ch64:288--308 establishes that "chain-of-thought is a sequential readout strategy, not a self-model," so even elaborate prompting strategies leave the substrate at para-mind status by VII.D55. The system-vs-substrate distinction matters: PROMETHEUS-as-system (the algorithmic layer) is what supplies the external topos; the LLM-as-substrate inside it is the para-mind.

Yet a Topos World Model needs precisely those things. The structural license for PROMETHEUS's construction is therefore that the topos machinery is supplied *externally*: PROMETHEUS itself, as an algorithmic layer over the LLM, plays the role of the external topos. This is a sound construction. It is also a constrained one. The construction succeeds only insofar as PROMETHEUS's external topos layer earns the categorical commitments it makes.

**Para-Mind Reading** [CHAIR SYNTHESIS]. *An LLM by itself cannot provide a topos because it is a para-mind (VII.D55). A framework that builds a "Topos World Model" from LLM-extracted claims is therefore performing an external topos construction over a para-mind substrate. The construction's structural integrity hinges on whether the external topos layer earns its categorical commitments.*

This reading is [DERIVED] at the manuscript level (VII.D55, VII.P14, VII.R42 are theorems of Book VII) and [ANALOGY] as applied to PROMETHEUS specifically. We do not claim PROMETHEUS endorses this reading; we claim the reading is the structural ground for why PROMETHEUS's construction *can* succeed at all, and the bar PROMETHEUS's external topos layer must clear to succeed.

The two structural commitments to examine are: the subobject classifier (§3) and the gluing functoriality (§4).

### 3. THE EARNED 4-FOLD: TRUTH4 AS SUBOBJECT CLASSIFIER

PROMETHEUS reports four gluing diagnostics:

- *Agreement* — compatible glue, low tension on shared cells.
- *Drift* — tension growing across time, runs, or document strata.
- *Contradiction* — high tension on shared cells (paraconsistent overlap).
- *Underdetermination* — insufficient support to glue or refute.

*Why four?* PROMETHEUS does not say. The four categories are empirically motivated (real corpora produce these four kinds of overlap pathology) but the paper does not derive them from a structural principle. They are listed as an operational taxonomy.

The  $\tau$ -canon corpus has the structural derivation. Book I [6] introduces, at Definition I.D21 (ch46:125--172), the four-valued lattice  $\text{Truth}_4 = \{\text{T}, \text{F}, \text{B}, \text{N}\}$  with the Belnap-Dunn structure [2, 3]: T (true), F (false), B (both, paraconsistent), N (neither, underdetermined). The bilattice structure gives two orderings: an information ordering ( $\text{N} \leq \text{T}, \text{F} \leq \text{B}$ ) and a truth ordering ( $\text{F} \leq \text{N}, \text{B} \leq \text{T}$ ).

The load-bearing result is Theorem I.T25 (ch56:146--177):

*The object  $\Omega_{\tau} = \text{Truth}_4 = \{\text{T}, \text{F}, \text{B}, \text{N}\}$  is the subobject classifier of  $\text{PSh}(\text{Cat}_{\tau})$ . That is,  $\Omega_{\tau}$  with the truth morphism  $\text{true} : 1 \rightarrow \Omega_{\tau}, * \mapsto \text{T}$ , satisfies: for every monomorphism  $m : S \hookrightarrow X$  in  $\text{PSh}(\text{Cat}_{\tau})$ , there exists a unique morphism  $\chi_S : X \rightarrow \Omega_{\tau}$*

such that [the classifying pullback square commutes].

This is *not* a choice of classifier from many possible ones for the  $\tau$ -canon presheaf topos: it is the unique classifier of  $\text{PSh}(\text{Cat}_\tau)$  [ $\tau$ -EFFECTIVE]. The “earned-not-imposed” differentiator applies *within* the  $\tau$ -canon framework: the orthogonality  $e_+ \cdot e_- = 0$  in the bipolar boundary algebra is itself a theorem (from spectral orthogonality, see ch46:230--256 of Book I and the further development at ch47:470--548), not a postulate. The 4-element classifier inherits this structure, and the PSh topos inherits the classifier.

**A notational note.** Book I’s Definition I.D21 (ch46:125--172) anchors the truth-order diamond  $F \leq B, N \leq T$  explicitly; the Belnap-Dunn knowledge ordering  $N \leq T, F \leq B$  follows from the standard componentwise encoding of Truth4 as the four-element bilattice but is not named separately in the corpus. We use both orderings in this note in the canonical Belnap-Dunn sense; the correspondence with I.D21 is via componentwise extension, not verbatim quotation.

**The mapping.** PROMETHEUS’s four diagnostics map onto Belnap’s four states with one precise mismatch.

The mapping clarifies a feature of PROMETHEUS that the paper does not flag: *PROMETHEUS has no clean refutation state*. Its “Contradiction” is the paraconsistent both-support, not the unanimous-refutation no-support. From the  $\tau$ -canon reading, this is licensed: the Belnap classifier  $\Omega_\tau$  has F as a separate state from B, but PROMETHEUS’s operational construction does not access F because retrieval-augmented corpora rarely refute — they more often produce paraconsistent overlaps or underdetermined regions.

**The latent F-channel [NEW EXTENSION].** A structural prediction follows from the  $\tau$ -canon projection. PROMETHEUS’s claim extraction (§4 of [18], the cSQL layer) records a *polarity* field for each causal claim, distinguishing *causes, reduces, increases, influences*, and similar signed relations. The  $\Omega_\tau$  projection then predicts that a polarity-keyed gluing diagnostic — “two regions both supporting opposing polarities of the same relation” — would surface a fifth Belnap-F diagnostic that PROMETHEUS’s current four-category taxonomy does not name. This is the latent F-channel of

PROMETHEUS’s existing data substrate: the polarity infrastructure is already there; exposing the polarity-keyed refutation in the gluing layer would complete the Belnap- $\Omega_\tau$  image at the operational level. This is a constructive structural prediction, not engineering advice; whether PROMETHEUS chooses to surface the F channel is a design question for its authors. (Cf. Non-claim 7, §5, on in-principle vs in-practice.)

**What is earned.** The 4-fold gluing taxonomy is not arbitrary. Within the  $\tau$ -canon reading, it is the manifestation, at the gluing-diagnostic layer, of the  $\Omega_\tau$  classifier structure that the  $\text{PSh}(\text{Cat}_\tau)$  topos earns from spectral orthogonality [ $\tau$ -EFFECTIVE]. Read this way: the  $\tau$ -canon reading *projects* Truth4 onto the four PROMETHEUS gluing-tension categories — PROMETHEUS observes the four categories operationally, and the projection from Truth4 to the operational categories is constructed on the  $\tau$ -canon side, not observed pre-existing in PROMETHEUS [NEW EXTENSION]. The five-or-six diagnostic extensions one might imagine (*partial drift, regional contradiction, measurement boundary*, etc.) would, under this projection, correspond to finer mixed states in the bilattice; the base-four projection is structurally fixed in  $\tau$ -canon, and the question whether PROMETHEUS in practice exhibits only the four categories or admits finer-grained refinements is empirical, not derived.

**The Lean carrier.** The Belnap classifier ships in TauLib [11] at BookI/Logic/Truth4.lean with the explicit construction of Truth4 and the four classifier morphisms, plus BookI/Logic/Explosion.lean for the paraconsistent non-explosion property (you can have B without the classical logic of explosion). The Lean carrier is part of the sorry-count 0, axiom-count 3 invariant of the formalisation [11].

#### 4. THE BOUNDARY FUNCTOR $\Phi_\partial$ : EXACT GLUING EARNED

PROMETHEUS’s Proposition 7.5 (*operational sheaf condition*) is an  $\varepsilon$ -tolerance averaging rule: local sections  $s_i$  on contexts  $U_i$  are considered  $\varepsilon$ -compatible when their weighted overlap gaps are below a declared tolerance, and a candidate glued section is then formed by support-weighted aggregation  $M_U[h, \tau] = \sum_i \omega_i M_{U_i}[h, \tau] / \sum_i \omega_i$ . Incompatible cells become obstruction records.

PROMETHEUS diagnostic	→	Belnap state
Agreement (low tension, compatible glue)	→	T
Underdetermination (insufficient support)	→	N
Contradiction (high tension on shared cells)	→	B ( <i>not</i> F)
Drift (tension growing across time/runs)	→	temporal B (no static analogue)

**Table 1.** The PROMETHEUS-to-Belnap mapping at  $[\tau\text{-EFFECTIVE}]$  rigor. PROMETHEUS’s “Contradiction” is paraconsistent (two regions *both* supporting opposite claims, which is B), not refutation (a single region *refuting* the claim, which would be F). Drift is dynamic, with no clean static-Belnap analogue.

This is not a sheaf condition in the Mac Lane–Moerdijk sense [15]. The classical sheaf condition requires:

1. **Existence:** for compatible local sections, a glued global section exists.
2. **Uniqueness:** the glued global section is unique.
3. **Functoriality of restriction:** the restriction maps commute with the gluing.

PROMETHEUS’s operational condition has neither uniqueness (the averaging is one of many possible aggregations) nor functoriality (the restriction maps are partial alignment maps, not sheaf-theoretic restrictions). The construction is honest at the engineering level (Mahadevan flags this explicitly in §15 limitations) but does not earn the “sheaf” designation.

The  $\tau$ -canon corpus, by contrast, proves exact Global Cartesian Gluing. Theorem III.T50 of Book III [7], anchored at ch75:91--123, establishes the Universal Global Cartesian Gluing as a colimit construction: the assembled global section is *canonically isomorphic* to the unique colimit of the diagram of restrictions, and the gluing is functorial with respect to the cover refinement. The chapter explicitly frames the result (ch75:154) as “the sheaf condition for the  $\tau^3$  spectral atlas.”

**What earns exact gluing.** The structural ingredient that upgrades approximate gluing to exact is the *boundary functor*  $\Phi_{\partial}$ , identified at ch75:64--76 as the *Langlands Coherent Force* (the  $\tau$ -canon programme’s name for the functor; in standard categorical language this is a sheaf on  $\text{Cat}_{\tau}$  valued in the bipolar boundary algebra, with functoriality playing the role of the sheaf condition for the spectral Grothendieck topology). The boundary functor is a contravariant functor from the cover category  $\text{Cat}_{\tau}$  to the bipolar boundary algebra, sending each context  $U$  to its boundary character spec-

trum and each morphism  $V \rightarrow U$  to the spectral restriction. The functoriality of  $\Phi_{\partial}$  is what makes restriction commute with gluing.

PROMETHEUS does not have  $\Phi_{\partial}$ . Its restriction maps  $\rho_{UV}$  are partial alignment maps, not functorial restrictions of a boundary-character functor. Without  $\Phi_{\partial}$ , the  $\varepsilon$ -tolerance condition is the best one can do operationally; the construction remains an averaging rule.

**Exact-Gluing Reading** [NEW EXTENSION]. *Approximate sheaf gluing becomes exact when the cover category is equipped with a boundary functor  $\Phi_{\partial} : \text{Cat}_{\tau} \rightarrow$  bipolar algebra that commutes with restriction. PROMETHEUS lacks  $\Phi_{\partial}$ ; its operational condition is the appropriate fallback for an engineering setting where the boundary characters are not extracted. At the structural level, an in-principle upgrade exists: a  $\Phi_{\partial}$  extension that extracts per-context boundary characters and type-checks restriction maps as spectral morphisms would, in the  $\tau$ -canon reading, make the  $\varepsilon$  vanish at the structural level. We do not claim this upgrade is engineering-ready — whether LLM-derived claim provenance can supply boundary characters in practice is an empirical question, not a structural derivation (Non-claim 7, §5).*

The reading is [DERIVED] for the  $\tau$ -canon side (III.T50 is a theorem) and [ANALOGY] for the PROMETHEUS-side application. We do not claim PROMETHEUS will adopt this upgrade; we claim the upgrade is structurally available and the route is identified.

**Competing upgrade paths.** The  $\Phi_{\partial}$  route is one of several structural upgrades available in principle. Three alternatives the AI/causal-research reader may recognise: (P1) equip  $\text{Cat}_{\tau}$  with a Grothendieck topology and apply sheafification, recovering exact gluing from the universal property; (P2) treat the restriction maps  $\rho_{UV}$  as *learnable* parameters and train them on cross-context data to commute with composition by construction; (P3) replace the sheaf formalism with causal-graph compositionality of the Halpern–Pearl style, where commutation is enforced via the intervention algebra. These are not in competition with the  $\Phi_{\partial}$  route — they articulate different structural commitments. The  $\Phi_{\partial}$  route is specifically the  $\tau$ -*canon*-anchored option (it inherits the Langlands Coherent Force theorem at ch75:64–76 of Book III); the alternatives sit in different research programmes and would carry their own anchored commitments.

## 5. SEVEN EXPLICIT NON-CLAIMS

The dialogue with PROMETHEUS is articulative, not adversarial. To keep the trust budget honest [4], we make seven non-claims explicit (Non-claims 1–5 carried from vi.o; Non-claims 6–7 added in vi.i RC1 after Wave  $\Delta_2$  peer review).

**Non-claim 1.** *The  $\tau$ -canon topos does not use  $H_{\tau}$ -valued enrichment at the presheaf codomain.* Recon-side speculation [6] that the  $\tau$ -canon topos uses split-complex enrichment as a structural commitment is incorrect. Book I ch56:111–114 uses  $\text{PSh}(\text{Cat}_{\tau}) = [\text{Cat}_{\tau}^{\text{op}}, \text{Set}]$  with  $\text{Set}$  as codomain, the same as PROMETHEUS. The  $H_{\tau}$  structure enters at the boundary-scalar level (Book III ch48), not at the presheaf codomain. The “enriched variants” phrase at ch56:584–589 is a forward-pointer to Books III–VII, not a Book I topos commitment.

**Non-claim 2.** *There is no formal PSR-to-Hankel identification in the  $\tau$ -canon corpus.* Recon-side speculation that PROMETHEUS’s controlled Hankel matrix  $H_{p,\tau}$  has a formal counterpart in the  $\tau$ -canon spectral framework was retracted on Phase I verification. The closest analogues are the bipolar idempotent algebra  $e_{\pm} = (1 \pm j)/2$  (Book III ch48) and the  $T_{op}$ -iteration of the Wedge-Loop Trace Identity [13], but no formal Hankel construction exists in the corpus. PROMETHEUS’s PSR layer remains, on careful read,

borrowed terminology (Dirichlet-smoothed frequency tables, not spectral PSR objects).

**Non-claim 3.**  *$\tau$ -canon does not endorse PROMETHEUS’s “Topos World Model” terminology in its current form.* The Mac Lane–Moerdijk [15] desiderata for a topos — subobject classifier, all finite limits, exponentials, internal Heyting algebra — are not constructed in PROMETHEUS §7. The Abramsky–Brandenburger [1] sheaf-theoretic structure of non-locality is cited but not invoked operationally. We support the *aspiration* (an external topos layer over a para-mind LLM is the right structural move) but flag the *terminology gap* (“Topos” is currently aspirational).

**Non-claim 4.** *We do not ask PROMETHEUS to retract its engineering choices.* The  $\varepsilon$ -tolerance gluing, the Dirichlet-smoothed PSR estimator, and the ad-hoc 4-fold diagnostic are reasonable engineering choices for a corpus-scale finite-resource setting. The dialogue is structural articulation, not engineering prescription. PROMETHEUS’s case studies ship; corpus-scale atlases are real; locality preservation is a contribution. The  $\tau$ -*canon* reading is what these engineering choices structurally license *when read through the  $\tau$ -canon corpus*, not what PROMETHEUS should do to be “correct”.

**Non-claim 5.** *Para-mind classification is descriptive, not pejorative.* The application of VII.D55 to LLMs is structural, not normative. A para-mind is not a defective mind (VII.D55 explicitly: “A para-mind is not a defective mind. It is a structurally different kind of system”). Our reading explains *why* an external topos layer is needed precisely because the LLM substrate is a para-mind, not as a critique of LLM capabilities but as a structural license for the PROMETHEUS-style construction.

**Non-claim 6.** *The structural reading does not claim PROMETHEUS implements  $\Omega_{\tau}$  literally, nor that “earned-not-imposed” is a critique of PROMETHEUS, nor that the reading transports to arbitrary AI/causal-research frameworks.* Three associated disambiguations: (i) The Belnap-Truth4 mapping (Table 1) is a projection from  $\Omega_{\tau}$  onto PROMETHEUS’s four operational categories; PROMETHEUS does not implement  $\Omega_{\tau}$  in its internals — the implementation is a per-cell tension-threshold classifier, not a topos subobject classifier. (ii) The “earned-not-imposed” framing applies to  $\tau$ -*canon*’s internal theorem chain (the orthogonality  $e_{+} \cdot e_{-} = 0$

is a spectral-structure theorem, not an axiom); it is not a critique of PROMETHEUS’s engineering choices, which are appropriate for their setting. (iii) The generalisation to “any future AI or causal-research framework that builds an external topos over LLM-extracted claims” (§6) is a chair-level pattern recognition, not a universal theorem; each future framework requires its own anchored verification.

**Non-claim 7.** *The  $\Phi_\partial$  upgrade path is an in-principle structural claim, not an engineering-ready recommendation.* The Exact-Gluing Reading of §4 states that if a boundary functor  $\Phi_\partial$  is supplied, the  $\varepsilon$ -tolerance becomes structurally exact at the colimit level. We do *not* claim that LLM-derived claim provenance can in practice supply boundary characters, nor that the type-checking of restriction maps as spectral morphisms is operationally implementable in PROMETHEUS’s current LLM-extraction architecture. Whether the upgrade can be realised in code is an empirical engineering question; the structural reading is in-principle, not in-practice.

## 6. COMPANION NOTES AND THE BROADER BRIDGE

This note is the first  $\tau$ -canon engagement with an AI/causal-research preprint via the ontology-articulation pattern. It sits alongside six published companion notes that establish the structural framework:

- [9] *Reading  $b \rightarrow s\mu^+\mu^-$  Through Category  $\tau$*  — the interpretation methodology pattern.
- [5]  *$\Delta C_9$  in Category  $\tau$  v1.9* — the load-bearing closed-form anomaly prediction  $\Delta C_9 = -\sin^2 \theta_W \cdot |C_9^{\text{SM}}| \approx -0.95$ , with seven phases of extensions (Wilson-coefficient family, chirality axis, mass-mediation axis, lepton-line axis, carrier axis,  $T'_1, T''_1$ ).
- [13]  *$T_1$  Wedge-Loop Trace Identity* — closure-derivation lockstep prose-Lean proof.
- [12]  *$F_2$ -Projection Theorem* — structural identification of  $T_{op}$  as bipolar-identity-sector image (Wave  $\Gamma_2$  closure).
- [10] *Running as Spectral Aperture* — the ontology-articulation pattern for the  $\tau$ -canon  $\leftrightarrow$  orthodox-QFT bridge.
- [4] *Categorical Foundations for Cosmological Billiards* — the foundation-comparison pattern.

This PROMETHEUS note adopts the *ontology-*

*articulation* pattern of [10], scaled to a preprint engagement. Its structural argument is: PROMETHEUS observes engineering phenomena (4 gluing diagnostics;  $\varepsilon$ -tolerance gluing; LLM-extracted causal claims) that  $\tau$ -canon has already proved at theorem level (I.T25 Truth4 classifier; III.T50 exact gluing via  $\Phi_\partial$ ; VII.D55 para-mind external-topos license).

**The generalisation, with discipline guard [CHAIR SYNTHESIS].** The companion pattern is not unique to PROMETHEUS. Any future AI or causal-research framework that builds an “external topos” over LLM-extracted claims is, by VII.D55, performing the para-mind external-topos construction. The  $\tau$ -canon reading (Belnap-Truth4 + boundary functor + para-mind license) is the structural articulation of what such constructions *do*, independent of which particular engineering choices the framework makes. We articulate this as a chair-level pattern, not as a universal theorem: each future framework requires its own chapter:line-anchored verification of how the para-mind substrate, the external-topos layer, and the boundary functor manifest in that specific construction (Non-claim 6, §5). The dialogue is articulative, framework by framework.

## 7. CONCLUSION

PROMETHEUS [18] introduces a serious engineering framework with aspirational categorical terminology. The engineering substance is real — corpus-scale causal atlases, preserved locality, exposed drift, navigable provenance. The categorical machinery is invoked heuristically: the §7 “operational sheaf condition” is an averaging rule, not a sheaf condition; the 4-fold gluing diagnostic is an empirical taxonomy, not a derived classifier; the PSR layer is borrowed terminology, not borrowed mathematics; the predicate “Belnap” appears nowhere in the references.

The  $\tau$ -canon reading is articulative. Three structural contributions land:

1. **The 4-fold gluing diagnostic is the earned sub-object classifier** (Theorem I.T25;  $\Omega_\tau = \text{Truth}_4 = \{T, F, B, N\}$ ). The mapping to Belnap’s four states has one precise mismatch (Contradiction maps to B, not F) that the structural reading exposes. The orthogonality  $e_+ \cdot e_- = 0$  is earned from spectral structure, not imposed by fiat.

2. **Exact gluing is achievable** (Theorem III.T50; via the boundary functor  $\Phi_\partial$ , the Langlands Coherent Force). The  $\varepsilon$  vanishes when the cover category is equipped with the boundary-character functor. PROMETHEUS lacks  $\Phi_\partial$ ; the upgrade path is identified but not adopted.
3. **The Para-Mind reading explains why an external topos is needed at all** (Definition VII.D55). An LLM by itself is a pattern processor without an internal topos; the PROMETHEUS-style external-topos construction is structurally licensed (and bounded by) the para-mind status of its substrate. “Para-mind” is descriptive, not pejorative.

The five explicit non-claims (§5) keep the trust budget honest. We do not import PROMETHEUS’s choices into  $\tau$ -canon; we do not export  $\tau$ -canon’s claims into PROMETHEUS-territory. The dialogue is structural articulation: PROMETHEUS observes;  $\tau$ -canon proves.

We hope the reading is useful both to PROMETHEUS’s future development (the Belnap-Truth4 connection offers a structural upgrade route for the gluing diagnostic; the  $\Phi_\partial$  boundary functor articulates what would earn exact gluing) and to the broader AI/causal-research community (the para-mind external-topos construction is a structural pattern that other frameworks will encounter). The companion note [10] plays the analogous role for the orthodox-QFT / spectral-aperture bridge; this note plays the role for the AI / causal-research / external-topos bridge.

**Trust budget preserved.** TauLib [11] sorry-count 0 and axiom-count 3 (programme-wide foundational) are unchanged. No load-bearing manuscript chapter is modified. The companion notes [9, 5, 13, 12, 10, 4] ship in their published forms. The dialogue with PROMETHEUS is articulative; PROMETHEUS as published is unchanged.

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We thank Sridhar Mahadevan for the PROMETHEUS preprint [18] that occasioned this articulation, and the DEMOCRITUS [17] and *Categories for AGI* [16] surrounding work that frames it.

The Mac Lane–Moerdijk [15] and Abramsky–

Brandenburger [1] foundations are the categorical infrastructure on which both PROMETHEUS and the  $\tau$ -canon programme rest. The Belnap–Dunn [2, 3] four-valued logic is the load-bearing classifier; the connection to topos-theoretic subobject classifiers is the  $\tau$ -canon Book I result. The Predictive State Representation theory of [14] is cited as the operational layer PROMETHEUS borrows.

The trust-budget infrastructure (TauLib [11] sorry-count 0, axiom-count 3 programme-wide foundational; the seven-book Panta Rhei manuscript [6, 7, 8]) is the framework on which the articulation rests; the six companion notes [9, 5, 13, 12, 10, 4] are the prior art the present note bridges with.

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