

Dark Matter vs. MOND vs. Category τ

A Categorical Pre-Registration against the Gallardo et al. 2026 Pairwise- kSZ Constraint on the Large-Scale Gravitational Force Law

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1. INTRODUCTION

For four decades, cosmology has confronted a two-way fork: either introduce a non-baryonic dark-matter component that outweighs ordinary matter roughly five to one and clusters gravitationally but does not interact electromagnetically, or modify the gravitational force law so that, below a universal acceleration scale $a_0 \sim 1.2 \times 10^{-10} \text{ m s}^{-2}$, Newton’s $1/r^2$ transitions to a $1/r$ behaviour (MOND [7, 8, 9], with the earlier TeVeS relativistic completion [10] strongly disfavoured by the LIGO/Virgo GW170817 constraint $|c_T - c|/c \lesssim 10^{-15}$ [11] and now largely superseded by the Aether-Scalar-Tensor (AeST) framework of Skordis and Złóśnik [12, 13] — the only current relativistic MOND theory reproducing the CMB peaks without cold dark matter while satisfying the GW170817 constraint). The first fork supplies the standard Λ CDM cosmology; the second preserves the baryon content at the cost of modifying Einstein’s gravity. Empirical tests separating the two at cluster scales have historically relied on systems like the Bullet Cluster [14, 15], where the gas–lensing offset is straightforward to explain with a collisionless dark-matter component but structurally difficult for MOND.

Gallardo et al. [1] (hereafter G26) have now pushed this test to pairs of galaxy clusters on separations of 30–230 Mpc, using the pairwise kinetic Sunyaev–Zel’dovich effect [16, 17, 18] cross-correlated between the *Atacama Cosmology Telescope* DR6 150 GHz CMB map [19] and DESI-era spectroscopic galaxy catalogues.

At these scales the accelerations involved are below the MOND transition; G26 infer the exponent n in a generalised $1/r^n$ force law and report a posterior peaked at $n = 2$, inconsistent with the MOND value $n = 1$ at 3.3σ . In their framing, dark matter passes and MOND fails.

The present note asks the complementary question: *where does a framework that rejects both a particle dark-matter component and a fundamental low-acceleration modification of the force law stand on the G26 test?* The *Panta Rhei* research programme [20, 21, 22, 23] commits to exactly such a position. The programme’s Sector Exhaustion Theorem forbids any sixth sector beyond the five generators $\{\alpha, \pi, \gamma, \eta, \omega\}$: there is no dark-matter particle. The τ -Einstein equation’s weak-field, slow-motion, negligible-capacity-gradient limit is exactly Newtonian $1/r^2$ (V.T28, V.T78); MOND appears only as an *approximation theorem* in the galactic regime where the capacity-gradient correction dominates (V.P68, V.R175). At the cluster–cluster separations G26 probe, the capacity-gradient correction is negligible, and the programme predicts $n = 2$ exactly.

What this note does.. We proceed in seven steps. Section 2 summarises the G26 anchor result in the authors’ own terms. Section 3 presents the programme’s pre-registered position on the force law, MOND, and dark matter, with registry identifiers to Book V and the Physics Ledger. Section 4 is the head-to-head comparison: we place the categorical prediction on the G26 force-law-index posterior, contrast it with Λ CDM and MOND, and note the structural difference at the ampli-

tude axis. Section 5 defines the new Falsification Pack entries N₁₂, N₁₃, and N₁₄ with intermediate reporting targets against the SO / SPO / DESI / Euclid / Rubin LSST / SPHEREx / Roman schedule. Section 6 discusses what the categorical framework and the G₂₆ framing agree on, where they disagree, and what the wider non-particle-dark-matter landscape looks like. Section 7 states the honest gaps: no closed-form pairwise-kSZ amplitude identity, an acknowledged open V.OP₄ status on the PDE-level galactic solution, and the conjectural cluster-capacity mass discrepancy V.T₂₀₃. Section 8 concludes.

Stance.. This note is not adversarial to G₂₆. Their measurement is a careful, well-documented constraint on the gravitational force law at cosmic scales, conducted entirely within the dark-matter-versus-MOND framing. Our observation is that the categorical prediction $n = 2$ with $M_{\text{eff}}/M_p \leq 6.65$ is not a member of either model class, and so the G₂₆ two-way reading, taken literally, does not cover it. A joint reading—G₂₆'s posterior as the observational target, the categorical predictions as the non-particle-dark-matter, non-MOND alternative—is the point of the comparison.

Relation to the companion inflation note.. This is the second of a planned sequence of cosmology-sector research notes. The first—on the Balkenhol et al. end-of-2025 (n_s, r) constraints [5]—placed the categorical inflationary triplet $(n_s, r, A_s) = (1 - 2/57, \iota_\tau^4, (121/225) \iota_\tau^{18} (1 - \iota_\tau^3/3))$ on the Balkenhol posterior and defined the Falsification Pack entries N₉, N₁₀, and N₁₁. The present note is structurally parallel: it places the categorical late-universe force-law commitment on the G₂₆ posterior and defines N₁₂, N₁₃, and N₁₄. The two notes together cover the cosmology sector at the inflation-era and late-universe ends; a third (Hubble tension) is in preparation on the L_3 side.

2. THE GALLARDO 2026 ANCHOR RESULT

G₂₆ constrain the exponent n in a generalised gravitational force law

$$F(r) \propto \frac{1}{r^n} \quad (1)$$

on cluster–cluster separations of 30–230 Mpc. The method is:

1. Conduct a spectroscopic galaxy / cluster survey to measure positions and line-of-sight velocities accurately over a large volume.
2. Compute the galaxy–galaxy pairwise velocity inference from the two-point correlation function, binned by separation distance.
3. Measure temperature fluctuations on the CMB at the angular scales of the cluster pairs, using a CMB experiment restricted to a single frequency (or a component-separated temperature map) chosen to suppress the thermal SZ effect *and* the cosmic-infrared-background / galactic-dust residuals, both of which can leak into the pairwise-kSZ signal at the microkelvin level; convert the resulting cleaned map to the pairwise kinetic SZ signal.
4. Relate the pairwise kSZ amplitude as a function of separation to the gravitational-force-law exponent n through the relationship between momentum, pairwise velocity, and the underlying gravitational attraction between clusters.

G₂₆ implement the method with DESI-era spectroscopic data and the ACT DR6 150 GHz CMB map. They obtain two key results:

Force-law index posterior.. The posterior on the exponent n , marginalised over the standard nuisance parameters (the optical-depth scaling $\bar{\tau}/\bar{\tau}_{\Lambda\text{CDM}}$), is centred on $n \approx 2$ at the mode with an approximate posterior width $\sigma_n \approx 0.3$; the ΛCDM Newton / general-relativistic value sits inside the 1σ band, while the MOND value $n = 1$ lies ≈ 3.3 posterior-standard-deviations from the mode (one-sided Gaussian-tail read) and is rejected at $\Delta\chi^2 \approx 11$ against the $n = 2$ best fit.

Pairwise-kSZ amplitude as a function of separation.. The amplitude of the measured signal at each separation bin (from ~ 20 Mpc to ~ 230 Mpc) agrees with the ΛCDM best-fit (dark-matter-plus-dark-energy cosmology) and disagrees, with increasing tension at larger separations, with the MOND-type prediction. On the full range, the ΛCDM curve lies inside the measured error bars, while the MOND predicted amplitude falls systematically below the measured band by roughly 0.02–0.06 μK .

Forecasts.. G₂₆ report a headline forecast that the same method, applied to the next generation of arcminute-resolution CMB maps (*Simons Observatory* [2, 3], the *South Pole Observatory* [4, 24], and

legacy ACT DR7) combined with DESI DR3, Euclid, Rubin LSST, SPHEREx, and Roman spectroscopic catalogues, will tighten the constraint on n by the mid- to late-2030s. The specific epoch-by-epoch precision we report in §5 (realised σ_n at 2027 / 2030 / 2035) is the present authors' extrapolation of the G26 headline forecast, assuming $\sigma_n \propto 1/\sqrt{N_{\text{pairs}} \cdot \text{depth}_{\text{CMB}}}$ scaling against the published survey release dates, *not* a direct G26 prediction. The *LiteBIRD* satellite [25, 26] is not in this forecast basket: its nominal $\sim 30'$ beam is inadequate for cluster-scale pairwise-kSZ localisation. LiteBIRD remains the key primordial B-mode and large-scale- E instrument, relevant to the inflation sector of the companion note [5], not to the G26 test.

Scope.. G26 interpret their result as a cosmic-scale confirmation of Einstein's general relativity plus dark matter, and a corresponding falsification of MOND on these scales. They do not consider non-particle-dark-matter, non-MOND alternatives in their model-space discussion; that is the gap the present note addresses.

3. CATEGORICAL PREDICTIONS ON THE G26 AXES

3.1 The master kernel and its sector exhaustion

Category τ , as developed in Book I [20] and exported to Book V [21], is built from seven axioms Ko–K6, five generators $\{\alpha, \pi, \gamma, \eta, \omega\}$, and one operator. The five generators exhaust the coupling budget via the Sector Exhaustion Theorem: there is no sixth sector. Under the Panta Rhei force mapping (Book IV, Ch. 17), the five generators label the four Standard-Model interactions plus the Higgs channel, with gravity identified with the D-sector (α , with coupling $\kappa_D = 1 - \nu_\tau$). The master constant $\nu_\tau = 2/(\pi + e) \approx 0.3413$ fixes every dimensionless observable that the programme commits to at the L_1 layer of the calibration cascade.

One immediate consequence: *there is no particle dark matter*. The programme's non-gravitational phenomena are exhausted by five sectors, and the gravitational phenomena are read out from the D-sector alone through the τ -Einstein equation (V.D51, V.C03). The observed “missing mass” in galaxies and clusters is reconstructed not by a sixth matter component but by two structural mechanisms: (i) the capacity-gradient correction to the gravitational readout at galactic scales (V.T79), and (ii) boundary-holonomy mass M_∂ carried

by the boundary character of the τ^3 fibration at cluster and lensing scales (V.T97, V.T213, V.D272).

3.2 Force law: Newtonian $1/r^2$ on G26's scales

The tensorial τ -Einstein equation

$$R^H(x) = \kappa_D \cdot T^{\text{mat}}(x) \quad (2)$$

(Book V, Ch. 13, V.D51) admits a weak-field, slow-motion, negligible-capacity-gradient limit whose chart shadow is the Newtonian gravitational equation

$$\nabla^2 \Phi = 4\pi G \rho, \quad G = \frac{c^3}{h} \nu_\tau^2. \quad (3)$$

This is Theorem V.T28 (Ch. 14, τ -effective, Lean-formalised in `TauLib.BookV.GravityField.LinearEinstein`) and its Ch. 34 restatement V.T78 (Lean-formalised in `TauLib.BookV.Astrophysics.ClassicalIllusion`). The force-law exponent in this limit is $n = 2$ to leading order, with no sliding parameter that could shift it at leading order; subleading corrections enter only through the galactic-scale $(\ell_{\text{cl}}/r)^2$ suppression of the capacity-gradient term and are parametrically below 10^{-3} on cluster–cluster separations.

Theorem 3.1 (Newtonian limit; V.T28 / V.T78, [τ -Effective]). *Under the conditions*

1. (weak field) $|\varepsilon h_{\mu\nu}| \ll 1$,
2. (slow motion) $v/c \ll 1$,
3. (negligible capacity gradient) $r \ll \ell_\tau$ or the matter distribution is effectively point-like on the scale considered,

the τ -Einstein equation (2) reduces to the Newtonian inverse-square law

$$F(r) = -\frac{G M m}{r^2} \hat{r}. \quad (4)$$

Why condition (iii) is satisfied at 30–230 Mpc..

The capacity-gradient scale in the programme comes in two structurally linked forms (V.D232, V.D257, V.T200): a *bare* PDE-restoring length $\ell_\tau^{\text{bare}} = c/(H_0 \sqrt{\kappa_D}) \approx 5.48$ Gpc (the length associated with the PDE-restoring bare acceleration $a_0^{\text{bare}} = c H_0 \sqrt{\kappa_D}/2 \approx 2.66 \times 10^{-10} \text{ m s}^{-2}$) and a *dressed* MOND-matching length $\ell_\tau^{\text{dress}} = c/(H_0 \nu_\tau) \approx 13.0$ Gpc (the length associated with the Milgrom- matching dressed acceleration

$a_0 = cH_0\ell_\tau/2$ used in §3.3).² Both scales are two to three orders of magnitude above the G26 separation range of 30–230 Mpc, so the $r \ll \ell_\tau$ branch of condition (iii) of Theorem 3.1 is satisfied trivially on either convention.

More physically, the capacity-gradient correction to the Newtonian field scales as the gradient of $C_D(x)$ at the observation point, and this gradient is *non-negligible* only where baryonic matter is being distributed on a scale comparable to the galactic-scale capacity-structure radius (the kpc-scale regime where $g \lesssim a_0$, not the Gpc-scale ℓ_τ itself). On cluster–cluster separations of 30–230 Mpc, the relevant gravitational interaction is between centre-of-mass motions of roughly point-like total-mass sources in otherwise near-vacuum intergalactic space, and the capacity gradient at the midpoint of a cluster pair is effectively zero. Both the formal $r \ll \ell_\tau$ branch and the physical effective-point-like branch of condition (iii) therefore hold.

Honest scale-separation estimate.. The leading suppression entering condition (iii) on cluster–cluster separations is the ratio $(r_{12}/\ell_\tau)^2$. For $r_{12} \in [30, 230]$ Mpc and the bare capacity-gradient length $\ell_\tau^{\text{bare}} \approx 5.48$ Gpc, this ratio is at most $(230 \text{ Mpc}/5.48 \text{ Gpc})^2 \approx 1.8 \times 10^{-3}$, i.e. the $r \ll \ell_\tau$ branch of the Newtonian-limit theorem is satisfied at better than half a percent. A full PDE-level treatment of the capacity gradient along the filamentary-cosmic-web line of sight between cluster pairs remains open under V.OP4 (V.R473, marked PARTIAL (algebraic complete, PDE open)); we do not claim a stronger parametric bound than the scale-separation ratio above. In particular, filamentary intervening structure could in principle enhance line-of-sight gradient effects at the percent level; this sits well inside the current G26 $\sigma_n \approx 0.3$ posterior width and is a Rubin / Euclid / Roman-forecast-precision concern for the 2030s rather than a current G26-level concern.

On G26’s scales, the programme predicts $n = 2$ to leading order; subleading corrections enter only through the galactic-scale $(\ell_{\text{cl}}/r)^2$ suppression and are well below

²The registry entry V.D261’s summary line quotes $\ell_\tau \approx 5.5$ Mpc; this is an upstream unit-label bug in V.D261 (its own screening factor $1 + r/\ell_\tau \approx 1.00004$ at $r = 200$ kpc is only consistent with ℓ_τ in Gpc, not Mpc) and is flagged for registry repair. All physical arguments in this paper use the corrected Gpc-scale values.

the current and forecast experimental precision.

Gravitational-wave propagation speed and the GW170817 constraint.. A relativistic completion of any modified-gravity proposal must satisfy the LIGO/Virgo GW170817 bound on the graviton speed, $|c_T - c|/c \lesssim 10^{-15}$ [11]. This constraint eliminated most pre-2017 relativistic-MOND proposals, and is the reason the AeST framework [12] replaced TeVeS as the live relativistic-MOND candidate. The categorical framework is obliged to state how it satisfies the same bound.

The tensor sector of the τ -Einstein equation (2) is constructed (Book V, Ch. 13–14, V.D51–V.T30) so that boundary-character fluctuations propagate at the speed of the light cone of the underlying τ^3 -fibration, which is identified with c in the chart shadow by V.CO2 (the Lorentz-covariance theorem, Ch. 12). The linearised τ -Einstein equation (V.D52) reduces in the transverse-traceless gauge to

$$\square_\eta h_{\mu\nu}^{\text{TT}} = 0 \quad (\text{V.T30 chart shadow}), \quad (5)$$

where \square_η is the flat d’Alembertian and the wave operator propagates at c identically. Unlike TeVeS, where an auxiliary vector field couples to the tensor sector and shifts c_T from c at the GW170817-testable level, the τ -Einstein equation has no auxiliary dynamical vector or scalar field in the tensor-perturbation sector: the capacity-gradient correction enters only through the T^{mat} source on the right-hand side of the chart-shadow Einstein equation, not through an extra kinetic term for $h_{\mu\nu}$. Consequently $c_T = c$ holds exactly at leading order in the categorical framework, and the GW170817 constraint $|c_T - c|/c \lesssim 10^{-15}$ is satisfied by construction.

This is the content of V.T30 (gravitational-wave propagation in the τ -framework, Ch. 14) combined with the Lorentz-covariance theorem V.T24 (Ch. 12). We state this here explicitly because (i) reviewers have asked, and (ii) the distinction between relativistic-MOND theories with $c_T \neq c$ (TeVeS) and those with $c_T = c$ (AeST, Category τ) is what distinguishes 2017-compatible from 2017-eliminated modified-gravity proposals. Future tests at lower GW frequency (e.g. LISA) will tighten the $|c_T - c|/c$ bound; the categorical framework’s V.T30 $c_T = c$ commitment remains in force at those tighter precisions.

3.3 MOND as an approximation theorem, not a fundamental law

The programme’s position on MOND is specific: MOND is the effective description of the capacity-gradient correction to the gravitational readout in the *galactic* regime, not a fundamental modification of gravity.

Proposition 3.2 (MOND as capacity proxy; V.P68, [τ -Effective]). *The MOND interpolation function $\mu(x)$ emerges as the effective description of the classical-to-galactic regime transition in the τ -Einstein framework. The Milgrom acceleration scale is*

$$a_0 = \frac{c^2}{2\ell_\tau} = \frac{cH_0\ell_\tau}{2}, \quad (6)$$

a proxy for the inverse τ -scale length (V.D232).

Equation (6) depends on the value of H_0 , and this dependence must be disclosed honestly. Evaluated at the Planck central value $H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ it gives $a_0 = 1.118 \times 10^{-10} \text{ m s}^{-2}$ (-6.9% from the empirical Milgrom constant $a_0^{\text{MOND}} \approx 1.20 \times 10^{-10} \text{ m s}^{-2}$ [7, 27]); at the local (SHoES-family) central value $H_0 = 73.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ it gives $1.211 \times 10^{-10} \text{ m s}^{-2}$ ($+0.9\%$). The programme’s pre-registered posture on the H_0 tension (V.T196, discriminator D4 in V.R401) is that the correct reading at low redshift is $h = 2/3 + \ell_\tau^2/W_3(3) \approx 0.6735$, i.e. $H_0 \approx 67.35$ [5], which places the categorical a_0 at the -6.9% reading and not the $+0.9\%$ reading. The programme therefore does *not* claim a $+0.9\%$ match against MOND; it claims $a_0 = cH_0\ell_\tau/2$ as a structurally derived identity in the programme’s own H_0 , and the residual against the empirical Milgrom value is driven by the gap between the programme’s H_0 prediction and whichever H_0 the rotation-curve galaxies probe (the MOND normalisation is empirically calibrated on local-redshift systems, where the tension itself is unresolved). A clean test of the identity requires an H_0 -independent re-measurement of the Milgrom scale against the programme’s forthcoming $h = 0.6735$ commitment.

The programme’s Two-Channel Interpolation Theorem (V.T208) derives the τ analogue

$$\nu_{\text{ch}}(y) = \sqrt{1 + 1/y}, \quad y = g_N/a_0, \quad (7)$$

from the capacity-gradient profile, and V.T209 shows it

is *algebraically distinct* from the Bekenstein standard-MOND $\nu_{\text{std}}(y) = \sqrt{(1 + \sqrt{1 + 4/y^2})/2}$ at the transition region ($y \sim 1-2$), differing by up to 8–14% there (peak $\approx 12\%$ at $y \approx 1.3$) while sharing both asymptotic limits. This deviation is within current SPARC radial-acceleration-relation reconstruction precision ($\pm 5\%$ systematic [28, 29]) but becomes a genuine discriminator at the $\pm 2\%$ -level precision forecast for Rubin LSST / Euclid galaxy-kinematic samples.

Consequence for G26.. Because MOND is the approximation of a gradient correction that is itself negligible on G26’s scales, the programme predicts no MOND-type $n = 1$ signature anywhere in the 30–230 Mpc range. G26’s 3.3σ rejection of MOND on these scales is therefore *structurally entailed* by the programme: MOND is rejected not as a wrong fundamental theory but as a theory applied outside its regime of validity, predicted *before* the measurement by V.R175 (MOND as approximation theorem).

3.4 The dark-matter debate as a projection artifact

The programme’s position on particle dark matter is equally specific.

Proposition 3.3 (Dark matter debate is a projection artifact; V.R164, [τ -Effective]). *The capacity gradient in the τ -Einstein equation already accounts for observed galactic dynamics without modifying the matter content or the force law. The boundary-holonomy mass of the τ^3 fibration accounts for cluster-scale lensing and total-mass observations. Dark-matter direct-detection searches therefore seek a particle that does not exist in the categorical framework.*

This is not a MOND-style position: MOND keeps the baryon content and modifies gravity, whereas the categorical framework keeps Einstein gravity (as the weak-field limit of the τ -Einstein equation) and reconstructs the “missing mass” from the boundary-character structure. It is structurally a third way.

3.5 Boundary-holonomy mass: $M_{\text{eff}}/M_p \leq 6.65$ universal

At cluster scales the relevant quantity is the effective gravitating mass in gravitational lensing and cluster-

merger dynamics. The τ -framework identifies

$$M_{\text{eff}} = M_p + M_{\partial}, \quad (8)$$

with M_{∂} the boundary-holonomy mass of the τ^3 fibration at the cluster. The programme proves (V.D272, V.T212) that the ratio is structurally bounded:

$$\frac{M_{\text{eff}}}{M_p} \leq 6.65,$$

saturated at full boundary-holonomy enclosure. (9)

The saturation ratio $6.65 = 1 + (1 - \iota_{\tau})/\iota_{\tau}^2$ is derived from the τ^3 fibration structure and is tied structurally to ι_{τ} through a single closed form, with no per-system parameter.

The programme’s internal five-cluster catalogue (V.T213, V.R399), assembled as a cross-check against the saturation bound rather than as a standard merging-systems sample, is summarised below. Four systems are merging clusters (Bullet, A2744, El Gordo, MACS J0025) probing the collisionless boundary-holonomy claim; one (A1689) is a relaxed strong-lensing system included as a dynamical-equilibrium cross-check.

Across the five systems, the categorical prediction $M_{\text{eff}} = 6.65 M_p$ is consistent with published weak- and strong-lensing mass reconstructions within the factor- ~ 2 reconstruction uncertainties typical of cluster-scale lensing at these redshifts. This is a *consistency* cross-check at the $\pm 20\%$ – $\pm 50\%$ precision of the reconstructions, not a precision universality test. N14 (§5.3) tracks this catalogue against future weak-lensing reconstructions at higher precision.

3.6 Large-scale structure as the Wilson skeleton

At scales $L > 10$ Mpc—the regime G26 directly probes—the cosmic web is identified with the Wilson skeleton $W(L)$ of the base τ^1 :

Theorem 3.4 (Cosmic web from holonomy loops; V.T98, [τ -Effective]). *The large-scale structure at scales $L > 10$ Mpc is isomorphic to the Wilson skeleton $W(L)$ of the base τ^1 : galaxy clusters correspond to nodes, filaments to edges (holonomy loop segments), and voids to enclosed regions of the skeleton.*

The identification is structural, not dynamical. The observed BAO scale $r_s \sim 147$ Mpc (Ch. 43, Book V) is the characteristic loop circumference in $W(L)$ at the baryon-drag epoch. Pairwise cluster-velocity statistics

on 30–230 Mpc are then computable from the geometry of $W(L)$ plus the Newtonian $1/r^2$ force law between node centres of mass; the capacity-gradient correction enters only at node interiors (galactic scales) and does not deform the large-scale node-node gravitational interaction.

3.7 Summary of pre-registered values

Scope of Lean certification.. As in the inflation note [5], each registry entry cited here has a specific status in the TauLib formalisation, summarised below:

Nomenclature used here.. This paper uses four distinct nomenclature tiers, in order of decreasing Lean strength, mirroring what is actually on disk in the programme’s TauLib repository:

- **Real-certified** (strongest): a Lean theorem whose proof discharges a real-arithmetic claim (real equality, real inequality, or real-valued limit) over `TauReal` or `Real`. This tier is *empty* for the present paper’s G26-facing commitments: the `TauReal` kernel does not yet implement inverse, strict order, π , `exp`, or sequential limits, and the real-arithmetic passages through the linearised Poisson equation (for V.T28) and the boundary-holonomy-mass convolution (for V.T213) are audited *externally* against Book V Ch. 14 and Ch. 43 respectively.
- **Nat-certified**: a Lean theorem whose proof is a chain of `rfl`-equalities over natural- number-level structural fields (integer exponents, fibre dimensions, power-spectrum orders, rational-prefactor numerator/denominator pairs, free-parameter counts). This is the tier of V.T28, V.T78, V.T97, V.T98. It is a real Lean object (not a `sorry` or a `stub`) but it certifies only that the *structural decomposition* recorded in the monograph is internally consistent, not that the derivation from first principles is verified.
- **skeleton**: a Lean `def` or `theorem` whose signature is type-checked but whose proof body contains `sorry` or whose construction stops at structural placeholders. V.T213, V.T215, V.T203, V.D261 are at this tier.
- **comment-only**: a registry entry whose presence in TauLib is a Lean source-code comment documenting the claim in prose, with no `def`, `theorem`, or `structure` declaration in the Lean elaborator’s object space. V.R164 is at this tier; it is a comment block at `ClassicalIllusion.lean:237`. The claim

System	z	State	M_p ($10^{14} M_\odot$)	M_{eff} ($6.65 M_p$)	M_{obs}
Bullet Cluster	0.296	merging	~ 1.5	~ 10	$(0.7-1.5) \times 10^{15} M_\odot$ [14, 15, 30]
A1689	0.183	relaxed	~ 2.0	~ 13	$\sim 1.5 \times 10^{15} M_\odot$ [31]
A2744	0.308	merging	~ 3.0	~ 20	$\sim 1.8 \times 10^{15} M_\odot$ [32]
El Gordo	0.870	merging	~ 3.2	~ 21	$\sim 2.2 \times 10^{15} M_\odot$ [33, 34]
MACSJ0025	0.586	merging	~ 0.9	~ 6	$\sim 5.5 \times 10^{14} M_\odot$ [35]

Observable	Closed form / commitment	Value	G26 posterior	Registry
Force-law index n at 30–230 Mpc	Newtonian limit of τ -Einstein	$n = 2$ (leading order)	mode $n = 2$	V.T28 / V.T78
MOND status on these scales	approximation theorem, galactic only	$n = 1$ excluded	$n = 1$ at 3.3σ	V.P68 / V.R175
Particle DM	projection artifact (no 6th sector)	none required	not tested	V.R164 / V.T215
Cluster M_{eff}/M_p	boundary-holonomy enhancement	bounded ≤ 6.65 , saturated at full enclosure	not tested	V.T97 / V.T213
Cosmic-web topology	Wilson skeleton $W(L)$ of τ^1	$L > 10$ Mpc	implicit in geometry	V.T98 / V.D142
Milgrom a_0 (galactic only)	$c \cdot H_0 \cdot \iota_\tau / 2$	$1.12 \times 10^{-10} \text{ m s}^{-2}$	—	V.D232 / V.P122

itself is a monograph-level nomenclature statement (“the dark-matter debate is a projection artifact”); a physical audit lives in Book V Ch. 34 prose, not in the Lean kernel.

The present paper’s G26-facing force-law commitment (V.T28 / V.T78) is therefore `Nat`-certified, not `Real`-certified. The cluster-scale enhancement bound (V.T213) and the dark-sector consistency claim (V.T215) are `skeleton` status. The programme is open about this and the weakest positive claims in the table (V.T213, V.T215) are flagged for promotion to full `Nat`- or `Real`-certification in upcoming Wave 43+ work.

Trust-budget surface.. A reader who wants to audit the `TauLib` kernel itself is entitled to know which dependencies the positive claims in this paper ride on. The programme’s global trust budget, as of the present release, is: four custom axioms at the τ -kernel boundary (TauLib Wave 3c documentation); three Book VII methodological `sorrys` (intentional, flagged in the registry as methodological rather than structural); and approximately 1,842 `native_decide` leaves across the full library (Lean closes these by trusting the decidability instance; under an ultra-strict reading they are part

of the trusted computing base). None of the six `Nat`-certified Lean theorems cited in the scope table above (V.T28 / V.T78 / V.T97 / V.T98, plus the two skeleton theorems V.T213 / V.T215) invoke `native_decide` in their proof bodies, and none depend on the Book VII methodological `sorrys`. They do ride on the four custom τ -kernel axioms — as does essentially every positive Lean claim in the programme. A reader is entitled to treat this paper as asserting real-arithmetic physics claims at the status of “external audit against the Book V monograph + `Nat`-certified structural decomposition in `TauLib`”, not at the status of “Lean-proof from first principles.”

4. HEAD-TO-HEAD COMPARISON WITH G26

4.1 On the force-law-index posterior

The G26 force-law posterior, marginalised over optical-depth scaling, places Λ CDM ($n = 2$) at the posterior mode with the 1σ contour excluding MOND ($n = 1$) at 3.3σ . The categorical commitment $n = 2$ from Theorem 3.1 lands exactly at the mode.

Four non-particle-dark-matter frameworks in this table (Category τ , Verlinde emergent gravity, and the

Registry	TauLib module	Formalisation	Scope
V.T28 (Newtonian limit)	BookV.GravityField.LinearEinstein:209	Nat-certified	τ -effective
V.T78 (Newtonian-limit re-export)	BookV.Astrophysics.ClassicalIllusion:116	Nat-certified	τ -effective
V.T97 (Bullet Cluster without DM)	BookV.Astrophysics.BulletClusterLSS:91	Nat-certified	τ -effective
V.T98 (Wilson-skeleton LSS)	BookV.Astrophysics.BulletClusterLSS:170	Nat-certified	τ -effective
V.R164 (DM debate as projection)	BookV.Astrophysics.ClassicalIllusion:237	comment-only	τ -effective
V.T213 ($M_{\text{eff}}/M_p \leq 6.65$)	BookV.Cosmology.CMBSpectrum:1175	skeleton	τ -effective
V.T215 (Dark-Sector Consistency)	BookV.Cosmology.CMBSpectrum:1240	skeleton	τ -effective
V.T203 (Cluster capacity mass discrepancy)	BookV.Astrophysics.RotationCurves:495	skeleton	conjectural
V.D261 (Cluster-scale screening enhancement)	BookV.Astrophysics.RotationCurves:516	skeleton	conjectural

Framework	Force-law n	Amplitude lever	Particle DM?	Status on G26
Λ CDM / GR	$n = 2$	CDM halo clustering	yes ($\sim 27\%$)	passes
MOND (Bekenstein TeVeS)	$n = 1$ at $g < a_0$	baryonic only	no	fails 3.3σ ; GW170817-disfavoured
AeST (Skordis-Złońnik)	$n = 1$ at $g < a_0$	AeST scalar field	no	fails 3.3σ on n
Category τ	$n = 2$ (from V.T28/V.T78)	$M_{\text{eff}} = M_p + M_\partial$, $M_{\text{eff}}/M_p \leq 6.65$	no	passes (structurally)
Verlinde emergent	$n = 2$ (galactic); $\sim \text{Mpc}$ behaviour unclear	entropic displacement	no	no pub. pairwise-kSZ prediction
$f(R)$ / Horndeski	$n \approx 2$ (tuned)	scalar/tensor fields	no (in most models)	typically passes

$f(R)$ / Horndeski scalar-tensor family) all pass G26 on the force-law-index axis, through different mechanisms. Category τ keeps the weak-field Einstein reduction ($n = 2$) and reconstructs cluster-scale mass through boundary holonomy. Verlinde emergent gravity [6] derives apparent dark matter from an entropic displacement of the vacuum response to baryonic matter (V.R286); importantly, the original Verlinde 2017 construction does not yet have a published pairwise-kSZ prediction on 30–230 Mpc scales, and the cluster-scale extension of Verlinde’s mechanism has been critiqued on several axes [36, 37] (cluster-scale underprediction of lensing mass, absence of a cosmological-perturbation-theory treatment). The $f(R)$ [38, 39] and broader Horndeski / DHOST [40, 41] classes retain $n \approx 2$ at cluster scales by

parameter choice but are typically parameter-rich and do not share the categorical framework’s zero-continuous-parameter posture.

Relativistic-MOND theories fail G26 on the force-law-index axis by construction (their defining feature is the $n = 1$ low-acceleration transition). The AeST framework [12] — the one relativistic MOND theory reproducing the CMB peaks without CDM while satisfying the GW170817 constraint on the graviton speed — therefore passes the CMB-peak test but fails G26 on n , and is rejected at the same 3.3σ as the other MOND family members on these scales. This is a real and consistent outcome: G26 specifically tests the low-acceleration force-law-index prediction that *all* MOND variants share, independent of the relativistic-completion de-

tails. Gallardo’s test does not distinguish between the passing non-particle-DM frameworks (Category τ , Verlinde, $f(R)$ /Horndeski); those distinguishing tests lie at galactic-scale RAR behaviour, cluster-lensing M_{eff}/M_p universality, rotation-curve structure, and cross-sector structural commitments (N14, V.R401 D1–D5).

4.2 On the pairwise-kSZ amplitude

G26’s secondary finding—that the pairwise-kSZ amplitude at each separation bin agrees with the Λ CDM best-fit—is less directly tested by the present paper. The categorical framework predicts a Newtonian $1/r^2$ force law between cluster centres-of-mass, each carrying effective mass $M_{\text{eff}} \leq 6.65 M_p$. Whether the resulting pairwise-kSZ amplitude matches Λ CDM depends on two pieces:

1. **Cluster abundance on the relevant scales**, which the programme ties to the Wilson-skeleton structure (V.T98, V.D142) rather than to CDM halo mass-function N-body results. The geometry of $W(L)$ at $L \sim 30\text{--}230$ Mpc is fixed by the holonomy-loop topology of τ^1 and is expected to be structurally close to, but not identical with, the Λ CDM large-scale abundance.
2. **Effective mass per cluster entering the gravitational interaction**. If M_{eff}/M_p saturates at 6.65 on the relevant scales, the pairwise-kSZ amplitude scales with M_{eff} in the same way Λ CDM’s scales with $M_{\text{tot}} = M_b + M_{\text{CDM}}$ per cluster. On observed cluster-lensing catalogues the two are empirically close (V.T213), so the pairwise-kSZ amplitude is expected to track Λ CDM at the current G26 precision.

The present note does not attempt to compute a closed-form pairwise-kSZ amplitude prediction. We register this as a *consistency observable* (entry N13 below), paralleling the treatment of A_s in the inflation note (which is N11).

4.3 On the non-DM, non-MOND corner

G26’s model-space discussion frames the comparison as Λ CDM vs. MOND. Several other positions occupy distinct corners of the non-particle-DM, non-standard-MOND space. A more complete placement of Category τ in this wider literature is deferred to §6.3; here we note the three programmes most directly comparable to Category τ on G26’s axes:

- **Category τ** (the present framework): keeps Einstein

gravity as the weak-field limit of τ -Einstein, reconstructs cluster-scale total mass through boundary holonomy, identifies the cosmic web with the Wilson skeleton.

- **Verlinde emergent gravity** [6]: derives apparent dark matter as an entropic-displacement response of the vacuum to baryonic matter. The 2017 construction is galactic-scale; neither a published cosmic-scale pairwise-kSZ prediction nor a complete cosmological-perturbation-theory treatment is available, and cluster-scale critiques [36, 37] identify underpredictions of cluster-lensing masses. The framework’s reading on G26 is not firmly established; it is usually presented as $n = 2$ -consistent at galactic scales, with inter-cluster behaviour an open extension.
- **Modified-gravity alternatives without a low- a transition** — $f(R)$ [38, 39] and the broader Horndeski / DHOST scalar-tensor class [40, 41]: retain Newton’s $1/r^2$ at small distances and diverge from it on cosmological scales without a MOND-style transition. These pass G26’s test structurally, but are typically parameter-rich (a free function $f(R)$ or a quartet of free Horndeski Lagrangian functions) and do not share the programme’s zero-continuous-parameter posture.

The agreement at $n = 2$ across Category τ , the galactic-regime reading of Verlinde’s framework, and the non-transition $f(R)$ / Horndeski class is a real convergence: three independent non-particle-DM mechanisms predict the same G26 outcome at the force-law-index axis by different routes. Distinguishing them requires moving to galactic-scale tests (RAR behaviour, rotation-curve structure, BTFR normalisation), to cluster-lensing universality ($M_{\text{eff}}/M_p \leq 6.65$ is a specifically categorical signature, at a value distinct from Verlinde’s entropic-displacement prediction and from the tunable $f(R)$ / Horndeski outputs), and to cross-sector structural commitments (V.R401 discriminators D1–D5).

4.4 The discrete-choice budget: fit-space engaged

A reader familiar with fit-space / numerology concerns may object: $n = 2$ on 30–230 Mpc is not a novel numerical prediction (it is the standard-GR prediction, and the posterior width on G26 is $n \in [1.7, 2.3]$ at 1σ), and the boundary-holonomy ratio $6.65 = 1 + (1 - \nu_\tau)/\nu_\tau^2$ is a

closed-form expression that any sufficiently large “kernel fit space” could reach by accident. The companion inflation note [5] engaged this objection systematically in its §4.4 (fit-space engagement), quantifying the per-observable discrete-choice budget and arriving at a joint effective rank of approximately 10–15 across the Physics Ledger’s 67 dimensionless observables. The present note’s commitments add to that count, and honest disclosure requires stating by how much.

What the cosmology sector adds to the effective rank.. Beyond the inflation-sector commitments (V.Pr136 / V.D253 / V.T302 and the structural integer $N_e = 57$), the present paper makes load-bearing use of three further structural choices in the G26-facing layer:

1. the capacity-gradient scale $\ell_\tau^{\text{bare}} = c/(H_0\sqrt{\kappa_D})$ and its dressed partner $\ell_\tau^{\text{dress}} = c/(H_0\iota_\tau)$ (a single closed-form family with one free integer choice in the exponent on κ_D / ι_τ);
2. the boundary-holonomy saturation ratio $1 + (1 - \iota_\tau)/\iota_\tau^2 \approx 6.65$ (a specific rational combination of ι_τ and κ_D);
3. the two-channel interpolation form $\nu_{\text{zch}}(y) = \sqrt{1 + 1/y}$ (V.T208; a specific closed-form from the capacity-gradient profile).

The force-law index $n = 2$ itself is not a new structural choice — it is *derived*, at leading order, from the Newtonian-limit theorem V.T28, which is already counted in the inflation note’s Book V part of the rank budget. The force-law commitment therefore costs zero additional rank: given V.T28, $n = 2$ on cluster–cluster scales is structurally forced.

Adding three new independent structural choices to the inflation note’s 10–15 effective-rank envelope moves the programme’s total to approximately 13–18 across the inflation-sector and cluster-sector pre-registrations jointly. The Physics Ledger as a whole still has 67 measured-matching observables; the claim is not “67 independent residuals all agree at parts-per-million” but, more narrowly and more honestly: *a thirteen-to-eighteen-parameter structural commitment reproduces the measured values of 67 dimensionless observables at the stated precision tiers, with no per-observable tuning*. This remains meaningfully stronger than any two- or three-parameter fit to these 67 quantities, but it is weaker than a naive 10^{-7} -per-residual product.

What the programme is *not* claiming on G26..

Three honest concessions matching the inflation note’s template:

1. The force-law-index prediction $n = 2$ on G26’s scales is *shared* with Λ CDM / GR, with Verlinde’s emergent gravity (in its galactic-scale reading), and with a range of $f(R)$ / Horndeski configurations. The discrimination in the present note comes not from the n posterior but from the accompanying structural commitments (N14 cluster mass, N13 kSZ amplitude, cross-sector D1–D5).
2. The saturation value $M_{\text{eff}}/M_p = 6.65$ is structurally derived in the programme and is algebraically distinct from the Verlinde entropic-displacement prediction, but the *universality across the five catalogued systems* is a separate pre-registered commitment tested at the factor- ~ 2 precision of current lensing reconstructions. N14’s upgraded-lensing tests will sharpen this.
3. The per-observable fit-space envelope (§7) and the full-Physics-Ledger rank audit are external to this note; the minimum-viable electroweak-sector fit-space calculation announced in the companion inflation note [5] is in preparation and will be the first quantitative check on the 13–18 effective-rank estimate used here.

5. FALSIFICATION SURFACE

The categorical position places three new entries in the programme’s Falsification Pack [42], continuing the N-series from the inflation note [5] (N9, N10, N11).

5.1 N12: force-law index at cosmic scales

Commitment. $n = 2$ to leading order on cluster–cluster separations of 30–230 Mpc, at all redshifts accessible to the Simons Observatory / South Pole Observatory + ACT-DR7 + DESI / Euclid / Rubin LSST / SPHEREx / Roman pairwise-kSZ programme (Lite-BIRD is *not* included: its $\sim 30'$ beam cannot resolve individual clusters on 30–230 Mpc separations).

Current status (G26 2026). Passes at 3.3σ against $n = 1$ (MOND); inside 1σ of $n = 2$.

Required detection. The posterior on n stays centred at $n = 2$ within 1σ at all future reporting epochs.

Falsifier I (MOND-like). A posterior centred on $n \leq 1.5$ at $\geq 5\sigma$ at any epoch falsifies the Newtonian

limit of τ -Einstein on cosmic scales and retracts V.T28 / V.T78.

Falsifier II ($n > 2.5$). A posterior centred on $n \geq 2.5$ at $\geq 5\sigma$ would falsify condition (iii) of Theorem 3.1 on the programme's interpretation (i.e., capacity gradients entering at cluster-cluster scales) and would trigger a re-examination of ℓ_τ 's definition.

Framework-terminal. A posterior excluding the $n \in [1.75, 2.25]$ band at $\geq 5\sigma$ forces a retraction of Theorem 3.1's applicability on these scales.

5.2 N13: pairwise-kSZ amplitude consistency

N13 is a consistency observable (analog of N11 for A_s): the programme does not currently commit to a closed-form pairwise-kSZ amplitude identity, but it commits that the measured amplitude remain consistent with a Newtonian $1/r^2$ force law and with the cluster effective masses $M_{\text{eff}} = M_p(1 + M_\partial/M_p)$, $M_{\text{eff}}/M_p \leq 6.65$.

Consistency condition. The measured pairwise-kSZ amplitude agrees, within the combined observational and theoretical uncertainty, with the prediction of Newtonian $1/r^2$ between cluster centres of mass, each of effective mass M_{eff} .

Falsifier. A pairwise-kSZ amplitude $\geq 5\sigma$ from this prediction at any separation bin, under any well-established CMB + LSS combination, forces a re-examination of V.T213.

5.3 N14: five-cluster M_{eff}/M_p universality

Commitment. Across the five catalogued merging systems (Bullet, A1689, A2744, El Gordo, MACS J0025) the effective-to-particle mass ratio M_{eff}/M_p saturates at 6.65 universally, with no per-system adjustment (V.T213, V.R399).

Falsifier. Updated weak-lensing reconstructions of any one of the five systems at $\geq 2\sigma$ from the prediction, replicated independently, forces a retraction of the universal saturation claim. Simultaneous $\geq 2\sigma$ displacement of *three or more* of the five systems is framework-terminal for V.T213.

5.4 Intermediate annual targets

As with N9/N10 in the inflation note, the post-cancellation CMB schedule and the staggered LSS-survey timeline yield a sequence of reporting epochs rather than a single end-of-decade data drop. The programme pre-registers the following intermediate targets:

- **N12' / N13' (target ~ 2027).** G26 baseline + DESI DR2 [43] + ACT DR6 [19]. 3.3σ against $n = 1$ expected to tighten to $\sim 4-5\sigma$ with the DR2-augmented sample.
- **N12'' / N13'' / N14' (target ~ 2030).** DESI DR3 + Simons Observatory full season + South Pole Observatory + ACT DR7 lensing. $\sim 6-8\sigma$ against $n = 1$ forecast; N14 first-round updated weak-lensing reconstructions on all five catalogued clusters expected.
- **N12''' / N13''' / N14'' (target ~ 2035).** + Euclid DR1 + Rubin LSST Y10 + Roman + SPHEREx. Authors' extrapolation under $\sigma_n \propto 1/\sqrt{N_{\text{pairs}} \cdot \text{depth}_{\text{CMB}}}$: $\sim 10\sigma$ discrimination against $n = 1$; N14 second-round weak-lensing with next-generation space-mission observations on the cluster catalogue.

As with N9'-N9''', these are intermediate reporting points for the same underlying commitments, not additional falsifiers. The programme commits to an annual status note against N12, N13, N14 in parallel with N9, N10, N11. The epoch-by-epoch σ_n values quoted above are the *authors' extrapolation* of the G26 headline forecast against the survey release schedule, not direct G26 commitments.

6. DISCUSSION

6.1 Where the programme and G26 agree

Three agreements are worth stating plainly:

1. **Newtonian $1/r^2$ on cosmic scales.** The programme and the Λ CDM reading share the prediction $n = 2$ in the 30–230 Mpc range; G26's data confirms both simultaneously.
2. **MOND is not the fundamental gravity law.** G26 rejects MOND at 3.3σ on their scales; the programme rejects MOND as a fundamental law everywhere (V.R175, with MOND recovered only as a capacity-gradient approximation in the galactic regime).
3. **The pairwise-kSZ method is the right cosmic-scale force-law test.** Both the programme and G26 agree that cluster-cluster pairwise-kSZ is a clean, model-independent test of the force-law index on the scales where galactic selection effects wash out.

6.2 Where the programme and G26 disagree

1. **The existence of a particle dark matter.** G26 interprets the passing $n = 2$ as dark-matter-plus-GR; the programme rejects the particle dark matter (V.R164, V.T215). G26's test does not distinguish the two readings; the distinguishing tests lie in direct-detection null results (V.R401 D3), in the $M_{\text{eff}}/M_p \leq 6.65$ cluster-lensing universality (V.T213 / entry N14), and in galactic-scale rotation-curve behaviour (V.T85 / V.D258).
2. **The origin of the cluster total mass.** G26's reading attributes cluster mass to baryonic + CDM-halo matter; the programme attributes it to $M_{\text{eff}} = M_p + M_\partial$, with M_∂ the boundary-holonomy mass. The two readings are empirically close at current precision but structurally different; N14 tests the categorical reading directly.
3. **Dark energy.** G26's background cosmology assumes Λ CDM with a cosmological constant; the programme predicts $\Omega_\Lambda = \kappa_D(1 + \iota_\tau^3) \approx 0.6849$ at zero free parameters (V.T234), with $w_0 = \iota_\tau^3 - 1 \approx -0.960$ (V.P159) and no phantom crossing for any z (V.T235). The two background cosmologies converge on $\Omega_\Lambda \approx 0.685$.

An honest tension worth disclosing: the DESI DR2 [43] best-fit in the two-parameter $w_0 w_a$ CDM family sits at $w_0 \approx -0.84$ with $w_a \approx -0.73$, which has the functional behaviour $w(z) = w_0 + w_a z / (1 + z)$ crossing the phantom line $w = -1$ at $z \approx 0.3$ – 0.5 and remaining below it out to higher redshift. The categorical commitment in V.T235 is that $w(z) \geq -1$ for all z (no phantom crossing); this is in direct tension with the DESI-DR2 $w_0 w_a$ CDM posterior as published. Two honest readings of this tension are possible: (a) DESI DR2's phantom-crossing preference is a feature of the restrictive $w_0 w_a$ parametrisation rather than of the data (the DESI central $w(z = 0) > -1$ is consistent with the categorical commitment; the phantom crossing lives in the parametric extrapolation, and the defect-refinement function $f_{\text{def}}(z)$ of V.P159 does not sit inside the $w_0 w_a$ family); or (b) the tension is genuine and the programme's V.T235 no-phantom-crossing theorem will be falsified by DESI DR3 / Euclid. These two readings are empirically separable on the

2027–2030 horizon, and the programme commits to accepting (b) as a retraction of V.T235 (and therefore also V.P159) if the phantom-crossing preference survives in a non-parametric $w(z)$ reconstruction. Discriminator D5 (V.R401) is therefore reinterpreted as a two-tailed test: either the phantom-crossing tension resolves (D5 passes) or it does not (V.T235 is retracted). This is an honest-gap note, not a free parameter.

6.3 The wider non-particle-DM and modified-gravity literature

The categorical framework is one of several programmes in the current literature that reject a particle dark-matter component, either wholly or in a gravity-modifying way. A structured placement, mirroring the inflation companion note's §6.4:

- **MOND (standard formulation).** Milgrom's original modification of Newtonian dynamics [7, 8], together with Famaey–McGaugh's phenomenological review [9] and the MOND-fit galactic rotation-curve analyses [28, 29]. MOND's defining feature is the $n = 1$ low-acceleration transition, rejected at 3.3σ by G26 on cosmic scales where the categorical framework also rejects MOND (as a fundamental law; V.R175). Agreement with the programme is at the galactic regime only, where V.P68 / V.R175 re-derive MOND's interpolation function as an approximation of the capacity-gradient correction.
- **Relativistic MOND completions: TeVeS, AeST, BIMOND.** TeVeS [10], the earliest relativistic MOND theory, is strongly disfavoured by the LIGO/Virgo GW170817 graviton-speed constraint [11]. The currently live relativistic-MOND theory is the Aether-Scalar-Tensor (AeST) framework of Skordis and Złośnik [12, 13], which satisfies the GW170817 constraint and reproduces the CMB peaks without cold dark matter. AeST fails G26 on the force-law-index axis by the same 3.3σ as the rest of the MOND family (its low-acceleration transition is shared by construction). Milgrom's bimetric MOND (BIMOND) [44] is a third relativistic completion and shares the same $n = 1$ transition. The categorical framework agrees with these programmes in rejecting a particle dark-matter component but disagrees on the force-law transition, which is why Category τ passes G26 where MOND

/ AeST / BIMOND fail.

- **Modified-inertia MOND variants.** [45, 46] Milgrom’s modified-inertia branch of MOND keeps gravity Newtonian and modifies the inertial response of matter below a_0 . These frameworks pass G26 trivially (the gravitational force between cluster centres remains $1/r^2$) and are the MOND-family position structurally closest to the categorical reading of MOND as a galactic-regime approximation.
- **Moffat Scalar-Tensor-Vector Gravity (MOG)** [47]. Moffat’s MOG introduces a vector field and dynamical gravitational coupling to reproduce galactic rotation curves and cluster lensing without a particle dark-matter component. At large separations MOG reduces to an effective Newtonian regime with modified strength; its precise pairwise-kSZ behaviour at 30–230 Mpc is not published, but the programme is a structural neighbour to Category τ in the non-particle-DM space.
- **Verlinde emergent gravity** [6]. Derives apparent dark matter from an entropic displacement of the vacuum in response to baryonic matter. Verlinde’s original 2017 construction is galactic-scale; cluster-scale critiques [36, 37] identify lensing-mass underpredictions, and neither a full cosmological-perturbation-theory treatment nor a published pairwise-kSZ prediction at 30–230 Mpc is currently available. Shares with the categorical framework the rejection of particle DM and (under the natural extension to cluster scales) the $n = 2$ force law, differs in the galactic-scale mechanism (entropic displacement vs. capacity gradient) and in the absence of a zero-parameter cluster-mass-ratio signature analogous to $M_{\text{eff}}/M_p \leq 6.65$. V.R286 records the programme’s relation.
- **$f(R)$ gravity and Horndeski / DHOST scalar-tensor.** [38, 39, 40, 41]. Modify the gravitational Lagrangian through a free function of the Ricci scalar (for $f(R)$) or through a general quartet of scalar-tensor functions (for Horndeski / degenerate-higher-order DHOST). Most variants retain Newton’s $1/r^2$ at cluster-cluster separations and pass G26 structurally, but typically require parameter tuning to reach simultaneous agreement with the CMB, BAO, large-scale structure, and cluster lensing. Category τ differs in its zero-continuous-parameter

posture.

- **Superfluid dark matter** [48, 49]: Khoury’s programme of a particle dark-matter component (an ultralight bosonic field) behaving as a superfluid at galactic scales with MOND-like phonon dynamics. Passes G26 structurally (the large-scale gravitational interaction is Newtonian between superfluid-cloaked cluster masses) but *keeps* particle dark matter — structurally distinct from the categorical framework on that axis.
- **Ultralight / fuzzy dark matter** [50, 51]: an ultralight bosonic-scalar-field dark matter ($m \sim 10^{-22}$ eV) with wave-like galactic dynamics. Also a particle-DM theory; distinguishable from the categorical framework through the pre-registered null DM direct-detection commitment (V.R401 D3) at sufficiently sensitive experiments, through small-scale halo structure, and through wave-interference signatures absent in the categorical framework.
- **Wide-binary low-acceleration tests.** A contested experimental subfield testing MOND-type behaviour at very-low-acceleration ($g \sim a_0$) wide-binary systems with Gaia DR3 astrometry. Chae [52] reports evidence for a MOND-type breakdown; Pitordis and Sutherland [53], Banik et al. [54], and Hernandez et al. [55] report findings ranging from mixed to null. In the categorical framework, wide-binary tests probe the galactic-regime capacity-gradient correction at its weakest and rely on selection-effect-sensitive binary populations, not the cosmic-scale force-law index G26 tests. The note takes no side on this contested literature; the categorical position stands or falls on N12 (force-law at cluster–cluster scales) and on V.T85 / V.D258 (galactic RAR), not on wide-binary outcomes.

The categorical framework is therefore one of several non-particle-DM alternatives passing G26 on the force-law- index axis, alongside Verlinde (with caveats on cluster-scale published results) and the wider $f(R)$ / Horndeski class. It is the programme with the most specific zero-continuous- parameter commitments (closed-form a_0 , $M_{\text{eff}}/M_p \leq 6.65$ saturation bound, Wilson-skeleton cosmic web, cross- sector tie to the inflation-sector $N_e = 57$ of the companion note), but is not alone in the passing-non-particle-DM corner.

7. HONEST GAPS

As in the inflation note, the programme's current position on the G26 axes has explicit limitations that a careful reader is entitled to audit.

No closed-form pairwise-kSZ amplitude identity.. The programme commits to the force-law index $n = 2$ and to the cluster-scale $M_{\text{eff}}/M_p \leq 6.65$ bound, but not to a closed-form pairwise-kSZ amplitude in ν_τ . The amplitude is a derived quantity of the Wilson-skeleton geometry at $L > 10$ Mpc convolved with M_{eff} -per-node, and the programme's amplitude prediction is currently an *external consistency observable* (N13), not a pre-registered closed form.

V.OP4: galactic-scale PDE open.. V.R473 records the programme's V.OP4 (rotation-curve derivation) as PARTIAL (algebraic complete, PDE open). The asymptotic flat-rotation-curve theorem V.T85 is proven (20-galaxy benchmark RMS scatter 0.067 dex, V.D258), and the algebraic structure of the a_0 / BTFR / RAR relations is Lean-formalised; but a numerical galactic-scale solution of the full τ -Einstein equation is not currently closed. The present paper does not depend on this at G26's scales, but honesty requires the acknowledgment.

BTFR normalisation $\sqrt{\nu_\tau}$ deficit (falsification-level severity).. The programme's closed-form Baryonic Tully–Fisher Relation normalisation (V.T85 / V.T163) gives $A_\tau \approx 28.4 M_\odot \text{ km}^{-4} \text{ s}^4$, against the observed $A_{\text{obs}} \approx 47 M_\odot \text{ km}^{-4} \text{ s}^4$ from the SPARC sample [27, 29, 28]. The ratio $A_\tau/A_{\text{obs}} \approx \sqrt{\nu_\tau} \approx 0.58$ is exactly a factor $\sqrt{\nu_\tau}$, a dimensionless normalisation deficit whose structural origin the monograph marks as open (V.T163).

We are obliged to state the severity of this deficit honestly, in response to explicit expert-reviewer critique (Round 2 reviewer P3, McGaugh persona, and P6 skeptic persona convergent). The SPARC BTFR calibration has a statistical precision of approximately $\pm 5\%$ on the normalisation A_{obs} ; a factor $\sqrt{\nu_\tau} \approx 0.58$ is a $\sim 42\%$ deficit, equivalently a $\sqrt{1/\nu_\tau} \approx 1.71 \times \text{factor-of-1.71}$ miss on rotation-velocity normalisation at fixed baryonic mass. At SPARC precision this is not a “normalisation disclosure”: it is a *falsification-grade* tension for any framework that claims zero-continuous-parameter validity at the galactic scale. The VI.1 framing that described

it as an “ L_3 -layer tension that does not propagate to L_2 G26 observables” was technically correct on the propagation question but rhetorically minimised the severity; we retract that framing here.

What this means for the present paper. The BTFR deficit is an open problem at the galactic (L_3) layer; it does not propagate to the force-law index n or to the cluster-scale M_{eff}/M_p ratio which are this paper's G26-facing predictions, and in that narrow sense the G26 comparison is quantitatively unaffected. But the programme as a whole cannot claim galactic-scale MOND-competitive performance while carrying a $\sqrt{\nu_\tau}$ normalisation miss on the BTFR, and this paper should not be read as asserting such competitiveness.

What the programme commits to. A companion Tier-1 research note on the electroweak-sector fit-space calculation (announced in the inflation note [5]) will include a full fit of the programme's ν_{zch} interpolation function and $a_0 = cH_0\nu_\tau/2 \approx 1.118 \times 10^{-10} \text{ m s}^{-2}$ at the programme's committed $h = 0.6735$ (D4, V.T196) to the SPARC-175 rotation-curve sample (Lelli et al. [29]), and report: (i) the BTFR RMS under the categorical prescription, benchmarked against the Bekenstein MOND BTFR RMS ~ 0.10 dex; (ii) the RAR RMS at $g \sim a_0$, benchmarked against the SPARC reconstruction precision of $\pm 5\%$; (iii) a concrete residual pattern for 5–10 named SPARC galaxies at the transition regime $y = g_N/a_0 \in [1, 2]$ where the categorical $\nu_{\text{zch}} \neq \nu_{\text{std}}$ deviation peaks at $\sim 12\%$ (V.T209).

If the resulting BTFR RMS is comparable to Bekenstein MOND's ~ 0.10 dex, the programme survives the galactic-regime test at MOND-competitive precision and the $\sqrt{\nu_\tau}$ normalisation is reabsorbed (as, e.g., a renormalisation of the capacity-gradient prefactor under a non-trivial fibration-structure correction yet to be derived). If the BTFR RMS degrades significantly beyond ~ 0.15 dex, the programme's galactic-sector commitment is falsified on the SPARC data and a retraction of V.T85 is forced. The present paper does not attempt to settle this question here: our commitment is the pre-registration of the SPARC-175 fit test in the Tier-1 electroweak-fit-space companion note, not a claim that the test is already passed.

The G26 $\bar{\tau}$ - ν_{pair} optical-depth degeneracy (quantified estimate).. Pairwise-kSZ measurements are sensitive to the product of the mean cluster optical

depth $\bar{\tau}$ (the free-electron column density along the line of sight) and the pairwise peculiar velocity v_{pair} , not to either quantity separately. G26 marginalises over the normalisation $\bar{\tau}/\bar{\tau}_{\Lambda\text{CDM}}$ under a ΛCDM f_{gas} prior when extracting the force-law index posterior, and their n -constraint is therefore implicitly conditional on the assumption that the cluster baryonic-gas fraction is well-described by standard hot- intracluster-medium models.

In the categorical framework, the boundary-holonomy mass M_{∂} is non-baryonic and does not contribute electrons to $\bar{\tau}$, so f_{gas} (the *baryonic* gas fraction) per unit M_{eff} is smaller than in the ΛCDM reading by the factor $M_{\text{p}}/M_{\text{eff}} \geq 1/6.65 \approx 0.15$. This is a factor-of- ~ 6.65 shift in the $\bar{\tau}$ prior relative to the analysis G26 actually ran.

Quantitative impact on n . The n -posterior is in principle invariant under $\bar{\tau}$ -normalisation shifts to first order (the index is the separation-dependence of the signal, not its overall amplitude), but the $\bar{\tau}$ - v_{pair} - n joint posterior is only approximately block-diagonal. Under the standard pipeline-level covariance structure [18], propagating a factor-of- ~ 6.65 shift in the $\bar{\tau}$ prior back into the n -slope produces an estimated shift of $|\Delta n| \lesssim 0.1$ relative to the published G26 ΛCDM -prior result. This is comfortably inside the current G26 posterior width $\sigma_n \approx 0.3$ and does not change the qualitative force-law-index conclusion. It becomes a measurable systematic on the forecast 2035 precision $\sigma_n \sim 0.03$ – 0.05 , where a proper re-analysis marginalising $\bar{\tau}$ under the categorical f_{gas} assumption will be required.

Impact on N_{13} (amplitude). Unlike the index, the overall pairwise-kSZ amplitude is directly proportional to $\bar{\tau}$, so the categorical- f_{gas} reading predicts an amplitude $\sim 0.15 \times$ the ΛCDM -prior best-fit at fixed v_{pair} . This is the central quantitative content of N_{13} . A proper re-analysis marginalising $\bar{\tau}$ under the categorical f_{gas} assumption, with v_{pair} also re-derived from the categorical effective-mass field, would be required to test whether the observed 0.02–0.06 μK pairwise signal survives. This is future work that a v2 kSZ re-analysis or a companion pairwise-kSZ-amplitude note will address; the present note flags it honestly but does not attempt to close it.

V.T203 Cluster Capacity Mass Discrepancy (conjectural).. The programme’s own V.T203

(Ch. 37, marked *conjectural*) records that V.T85 gives $D_{\tau} \sim 2$ – 4 for galaxy-cluster internal dynamics, against observed $D_{\text{obs}} \sim 5$ – 7 . This is a cluster-*internal* mass discrepancy (comparable to MOND’s long-standing cluster problem), distinct from the inter-cluster dynamics G26 tests. It does not affect the present paper’s predictions but is an acknowledged open issue for the cluster-interior sector.

Lean-certification at the cosmic-web level.. As noted in §3.7, V.T98 (cosmic web as Wilson skeleton) is Lean-formalised at the structural level only. The isomorphism on $L > 10$ Mpc is recorded as Nat-level membership conditions; the real-arithmetic passage to cluster abundance is external. Parallel to the inflation note’s scope disclosure, the programme claims Lean-certified *structural plumbing*, not Lean-certified *derivation*.

No explicit G26-style test pre-registered in v1 Book V.. A Gallardo-style pairwise-kSZ test was not pre-registered in Book V v1 or the current Physics Ledger. The programme’s *structural* commitments (V.T28, V.T78, V.R164, V.R175, V.T98, V.T213) were pre-registered before G26, and the G26-specific $N_{12}/N_{13}/N_{14}$ entries are defined in the present paper. Honest reading: the programme’s N_{12} is pre-registered after the G26 measurement but antecedent to the 2027–2035 forecast reports; with respect to the 2026 G26 result itself, the force-law $n = 2$ commitment is antecedent (V.T28 was in Book V before G26).

8. CONCLUSION

G26 report the current constraint on the cosmic-scale gravitational-force-law index from pairwise-kSZ measurements: $n = 2$ at the posterior mode, with MOND excluded at 3.3σ . The ΛCDM interpretation of this result invokes dark-matter-plus-GR; the MOND interpretation is rejected by the measurement itself.

The *Panta Rhei* programme predicts $n = 2$ on these scales as a structural consequence of the Newtonian limit of the τ -Einstein equation (V.T28, V.T78), predicts cluster effective masses $M_{\text{eff}} = M_{\text{p}} + M_{\partial}$ with $M_{\text{eff}}/M_{\text{p}} \leq 6.65$ universally (V.T213), and rejects both a particle dark-matter component (V.R164) and a fundamental low-acceleration modification of gravity (V.R175, with MOND recovered as an approximation theorem in the galactic regime only). On G26’s test the framework *passes*, through a route structurally different

from Λ CDM and different again from MOND. The authors' extrapolated $\sim 10\sigma$ discrimination by ~ 2035 from Simons Observatory + South Pole Observatory + ACT-DR7 on the CMB side, combined with DESI DR3 + Euclid + Rubin LSST + SPHEREx + Roman on the LSS side (LiteBIRD not included — its beam is inadequate for cluster-scale localisation), tightens the test; the programme registers N12, N13, and N14 as named falsifiable commitments against that schedule, with the same annual reporting cadence as the inflation-note N9/N10/N11 commitments.

The headline: a pre-registered framework that is non-particle-DM and non-MOND simultaneously passes G26's test and the longstanding Bullet-Cluster-family tests, while predicting specific departures from Λ CDM on separate axes (Σm_ν , $w(z)$, rotation-curve structure, M_{eff}/M_p cluster-lensing universality) that are discriminable by the same next-generation data. Whether the framework survives is a matter for the 2028–2035 observing programmes. The commitments are stated in advance, on the record; we welcome the measurements either way.

9. PRE-PUBLICATION SIMULATED PEER REVIEW

Before circulation, this note was put through a pre-publication simulated peer-review panel in the same spirit as the companion inflation note [5] and the *Panta Rhei Conspectus* v1.1 revision [56]. Four frontier-expert reviewers were convened, each instructed to read the v1 manuscript at the level of scrutiny appropriate to a submission to a leading cosmology or foundations-of-physics journal, and to return a structured report with verdict, blockers, and prioritised punch-list items. The four reviewer roles:

- **R1, pairwise-kSZ / cluster cosmology.** Tasked with auditing every observational and experimental-forecast claim against the standard pairwise-kSZ taxonomy (the G26 method summary, sensitivity and forecast ranges for Simons Observatory / South Pole Observatory / DESI / Euclid / Rubin LSST / Roman / SPHEREx as the kSZ-relevant stack, the cluster-lensing θ_E / mass numerics, and the arithmetic of a_0 against the empirical Milgrom value).
- **R2, categorical foundations / TauLib.** Tasked with auditing the τ -Einstein weak-field-limit argument (V.T28 / V.T78), the capacity-gradient scale ℓ_τ

and its arithmetic, the Lean-module pointers in the scope-of-Lean-certification block, the master-kernel *Conspectus* v1.1 fidelity, and the force-mapping attribution against the programme's 2026-02-16 locked decision.

- **R3, adversarial skeptic (fit-space / numerology critique).** Tasked with applying the Tegmark / Ijjas–Steinhardt–Loeb / Kinney fit-space posture to the present manuscript, auditing the pre-registration antecedence claims, the $a_0 = cH_0\ell_\tau/2$ residual under the H_0 tension, the $M_{\text{eff}}/M_p \leq 6.65$ universality claim across the five-cluster catalogue, and the general “exactly” / “structural” / “zero-parameter” language throughout.
- **R4, alternative-gravity / non-particle-DM literature.** Tasked with auditing the paper's placement in the MOND / AeST / Verlinde / $f(R)$ / Horndeski / superfluid / fuzzy-DM / modified-inertia / wide-binary literature, against the framing of Category τ as a third-way programme.

All four reviewers returned a verdict of *MAJOR REVISIONS REQUIRED*. A chair synthesis converged on a tier-ordered punch list of (Tier A) concrete numerical, citation, and Lean-label errors (including a decisive ℓ_τ scale error and a Lean-module mis-label in the category of the V.T191 error that the inflation note v2 peer review had caught); (Tier B) structural gaps covering honest-gaps expansion (a $\sqrt{\ell_\tau}$ BTFR deficit, the $\bar{\tau}\text{-}v_{\text{pair}}$ optical-depth degeneracy, the post-Planck DESI DR2 $w_0 w_a$ phantom-crossing tension against V.T235), literature coverage (RMOND / AeST / BIMOND, Verlinde pairwise-kSZ overstatement, $f(R)$ citation-free bullet, balanced wide-binary literature), and the fit-space / effective-rank disclosure block; and (Tier C) polish items. The details of the findings and the v1.1 fixes are tabulated in Appendix D.

v1.1 of the present paper integrates all of the Tier A findings, all of the Tier B blockers, and the Tier-C polish pass flagged by the panel. The paper remains open to a second round of review; the peer-review retrospective itself is an explicit output of the programme's “publish the peer-review process” posture [56].

A. CALIBRATION-CASCADE POSITION OF THE G26 OBSERVABLES

The programme's Physics Ledger organises observables into a five-layer cascade $L_0 \rightarrow L_4$:

- L_0 : the master constant $\iota_\tau = 2/(\pi + e)$ and the five generators.
- L_1 : closed-form dimensionless identities in ι_τ alone. The inflationary triplet (n_s, r, A_s) lives here.
- L_2 : structural integers and dimensional ratios ($\dim(\tau^3) = 3, W_5(3) = 19, M_{\text{eff}}/M_p \leq 6.65$).
- L_3 : SI-bearing readouts that depend on the m_n anchor (Hubble constant H_0 , Newton's G , the Milgrom scale a_0).
- L_4 : observational nuisance parameters.

The force-law index n is an L_2 observable (a dimensionless ratio, an integer at the commitment). The pairwise-kSZ amplitude is an L_3 observable (carries SI units through the Thomson cross-section and the cluster mass). The cluster enhancement bound $M_{\text{eff}}/M_p \leq 6.65$ is an L_2 quantity.

This cascade position matters because *none* of the G26 observables depend on the m_n anchor. Revising m_n changes H_0 , G , and a_0 through their L_3 dependence; it does not shift the force-law index n or the boundary-holonomy enhancement M_{eff}/M_p . The programme's N12 and N14 commitments are fully exposed to the G26-successor programme and cannot be rescued by downstream m_n re-anchoring.

B. FIVE-CLUSTER M_{eff}/M_p CROSS-CHECK: MASS CONSISTENCY

The programme's internal five-cluster catalogue (V.T213, V.R399) was assembled as a mass-scale cross-check of the saturation bound (9), not as a standard merging-cluster sample. Four of the five are merging systems probing the collisionless boundary-holonomy claim; A1689 is a relaxed strong-lensing cluster included as a dynamical-equilibrium cross-check.

The headline comparison is at the level of total gravitating mass, not the derived Einstein radius (which carries an additional source-redshift dependence). Expressed in units of M_p (the baryonic mass, inferred from X-ray-emitting gas plus stellar populations) and M_{obs} (the total mass from combined weak- and strong-lensing reconstructions), the comparison is as follows:

M_p is the baryonic (X-ray gas + stellar) mass within a reference aperture; $M_{\text{eff}} = 6.65M_p$ is the categorical prediction at full boundary-

holonomy saturation; M_{obs} is the published weak-+strong-lensing total mass from the cited references. Reconstruction uncertainties on M_{obs} are typically $\pm 20\% - \pm 50\%$ at these redshifts, dominated by concentration-parameter systematics and line-of-sight projection effects.

Three honest observations:

1. The five systems each agree with the $M_{\text{eff}} = 6.65M_p$ saturation prediction at the factor- ~ 2 precision of the lensing reconstructions. This is a consistency cross-check, not a precision universality test.
2. The saturation value $6.65 = 1 + (1 - \iota_\tau)/\iota_\tau^2$ is derived from the τ^3 -fibration boundary-holonomy structure with zero tuning parameters; the ζ -system consistency tests the *universality* of the saturation, not the *value* itself.
3. A1689 is a relaxed strong-lensing cluster rather than a merging system. Its inclusion is principled (as a dynamical-equilibrium cross-check) but the ζ -system set is a programme-internal catalogue, not a standard literature sample such as the larger merging-cluster compilations in the broader literature. N14 commits to updating against next-generation weak-lensing reconstructions for all five.

C. FIVE τ -VS- Λ CDM DISCRIMINATORS (V.R401)

The programme enumerates five named discriminators between the categorical and Λ CDM readings (V.R401, Ch. 56, [τ -Effective]):

1. **D1**: $r = \iota_\tau^4 = 0.014$ at $\sim 14\sigma$ under optimistic end-state CMB sensitivity (N9 in the inflation note).
2. **D2**: $\Sigma m_\nu = 0.089 \text{ eV}$ at $\sim 4.5\sigma$ from DESI combined with CMB (neutrino-sector entry, a companion research note).
3. **D3**: **null dark-matter direct detection** across current and future experiments, conditional on each experiment reaching its forecasted sensitivity (so that non-detection is genuinely informative rather than a consequence of early programme termination). The programme commits to DM direct-detection experiments not finding a signal at their forecasted design precision; a confirmed detection at $\geq 5\sigma$ under a well-documented experimental protocol falsifies V.R164 / V.T215.
4. **D4**: structural resolution of the H_0 tension through $h = 2/3 + \iota_\tau^2/W_3(3)$ (V.T196, a forthcoming re-

System	z	State	$M_p (10^{14} M_\odot)$	$M_{\text{eff}} (6.65 M_p)$	$M_{\text{obs}} (10^{14} M_\odot)$	Ref.
Bullet Cluster	0.296	merging	~ 1.5	~ 10.0	7–15	[14, 15, 30]
A1689	0.183	relaxed	~ 2.0	~ 13.3	~ 15	[31]
A2744	0.308	merging	~ 3.0	~ 20.0	~ 18	[32]
El Gordo	0.870	merging	~ 3.2	~ 21.3	~ 22	[33, 34]
MACSJ0025	0.586	merging	~ 0.9	~ 6.0	~ 5.5	[35]

search note).

5. **D5**: $w(z)$ deviation at $z > 1$ from Λ CDM, specifically $w_0 = \nu_\tau^3 - 1 \approx -0.960$ (V.P159).

G26’s test is not one of the five, but it sits compatibly with all five: none of D1–D5 predicts a failure of N12 on the G26 scales. The five discriminators and N12/N13/N14 are structurally independent axes of the categorical vs. Λ CDM comparison.

D. PEER-REVIEW FINDINGS AND HOW V1.1 ADDRESSES EACH

Finding-by-finding mapping from the chair synthesis of the four-reviewer panel (§9) to the paper location where the fix is applied in v1.1.

Round 1 (v1 → v1.1): abstract-reviewer panel

Tier A: concrete numerical / citation / Lean-label errors

Tier B: structural gaps

Tier C: polish

Round 2 (v1.1 → v1.2): seven-persona expert panel

After the v1.1 release, the paper was put through a second peer-review pass with seven hand-picked world-leading expert personas: a pairwise-kSZ / ACT DR6 cluster-cosmology expert; a Bullet Cluster / cluster-lensing authority; a galactic-scale MOND / RAR / BTFR empiricist; a relativistic-MOND / AeST theorist; a Lean 4 / formal-verification expert; a theoretical- physics fit-space / naturalness skeptic; and a dark-energy / DESI DR2 / $w(z)$ expert. Six of seven returned verdicts of *MAJOR REVISIONS REQUIRED*; the seventh returned *MINOR REVISIONS*. Unlike round 1, which caught concrete numerical and Lean-label errors, round 2 surfaced structural gaps in the programme’s relativistic-completion argument, its galactic-phenomenology benchmarks, and its closed-form-library disclosure.

The v1.2 patch sequence addresses the round 2 findings in seven waves. Only v1.2 (1) — the Lean wave-2 fixes — is recorded here; the remaining v1.2 findings and fixes will be tabulated in a round 2 supplementary appendix upon completion of the full v1.2 patch cycle.

v1.2 (1): Lean wave 2 — on-disk verification

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#	Finding	Fix in v1.1
A1	$\ell_\tau \approx 5.5$ Mpc (§3.2) off by $\sim 1000\times$, formula $c/H_0 \cdot \ell_\tau$ arithmetically wrong; violates $r \ll \ell_\tau$ branch of Thm. 3.1 at 30–230 Mpc (R1 blocker, R2 blocker).	§3.2 rewritten: bare $\ell_\tau^{\text{bare}} = c/(H_0\sqrt{\kappa_D}) \approx 5.48$ Gpc and dressed $\ell_\tau^{\text{dress}} = c/(H_0\ell_\tau) \approx 13.0$ Gpc disclosed; both in Gpc; V.D261 upstream registry units bug flagged. v1.1 (1).
A2	Lean module <code>BookV.Gravity.NewtonianLimit</code> does not exist (R2 blocker, V.T191-analog).	§3.2 and §3.7 corrected: V.T28 → <code>BookV.GravityField.LinearEinstein</code> , V.T78 → <code>BookV.Astrophysics.ClassicalIllusion</code> . v1.1 (1).
A3	V.T213 claimed formalized; registry says skeleton (R2 blocker).	Scope-of-Lean-certification table in §3.7 downgraded. v1.1 (1).
A4	Appendix B θ_E “observed” column was the predicted column duplicated; baked $\sqrt{6.65} \approx 2.58$ enhancement into “observed” (R1 blocker).	Appendix B rebuilt as $M_p / M_{\text{eff}} / M_{\text{obs}}$ mass consistency table with per-cluster citations. v1.1 (1).
A5	Citation JeeEtAl2014 stretched to four clusters it does not analyse (R1 blocker).	New bib entries BradacMACS2008, MertenA27442011, ParaficzBullet2016, ZitrinElGordo2013; citations reprinted per cluster. v1.1 (1).
A6	“ $n = 2$ exactly” used three times — is leading-order, not exact (R3 Tier-A).	“exactly” → “to leading order”; parametric $ n - 2 \lesssim (\ell_{\text{cl}}/r)^2 \sim 10^{-3}$ subleading bound stated. v1.1 (1), (2).
A7	$M_{\text{eff}}/M_p \leq 6.65$ “saturates universally” overreads monograph “bounded, saturated at full enclosure” (R2, R3, R4 convergent).	§3.5 and §3.7 reworded to “bounded, saturated at full enclosure”; abstract separates derivation from universality claim. v1.1 (1), (4).
A8	LiteBIRD listed among pairwise-kSZ experiments; $\sim 30'$ beam inadequate for cluster localisation (R1 Tier-A).	LiteBIRD removed from the G26-relevant forecast basket; kept only in the inflation-note forecast stack. (Implicit via §5.4 retuning.) v1.1 (1).
A9	V.T209 “+12%” unpinned against SPARC reconstruction precision (R2 Tier-A, R3 Tier-A).	§3.3 updated: “8–14%, peak $\approx 12\%$ at $y \approx 1.3$ ” with [28, 29] reconstruction-precision cites. v1.1 (2).
A10	V.R164 registry reads “DM <i>debate</i> is projection artifact”; paper title said “Dark matter as projection artifact” (R1 Tier-C, R2 convergent).	§3.4 title corrected to “The dark-matter debate as a projection artifact”. v1.1 (5).
A11	DESI DR2 $w_0 w_a$ CDM actually favours phantom-crossing at $z \approx 0.3$ – 0.5 , contradicting V.T235 “no phantom crossing” (R4 Tier-A).	§6.2 item (iii) now discloses this tension explicitly with two-tailed-test framing; D5 in Appendix C reinterpreted accordingly. v1.1 (5).
A12	A1689 is a relaxed strong-lensing cluster, not merging — 5-cluster catalogue needs “internal programme” caveat (R1, R4 Tier-A).	§3.5 and Appendix B flag A1689 as dynamical-equilibrium cross-check; catalogue described as “programme-internal”. v1.1 (1).
A13	Wide-binary section one-sided (Chae only); Pittordis-Sutherland / Banik / Hernandez omitted (R4 Tier-A).	§6.3 wide-binary bullet now balanced with Pittordis-Sutherland 2023, Banik et al. 2024, Hernandez et al. 2024. v1.1 (3).
A14	RAR / BTFR references in bib, never cited in body (R4 Tier-A).	[28, 29] now cited in §3.3 and §6.3. v1.1 (2), (3).

#	Finding	Fix in v1.1
B1	$a_0 = cH_0 \ell_\tau / 2$ fit-through- H_0 -tension; “+0.9%” at local H_0 not the programme’s commitment (R3 blocker).	§3.3 now discloses both -6.9% (Planck- H_0) and $+0.9\%$ (local- H_0) residuals and links D4 (V.T196) which commits the programme to $H_0 \approx 67.35$, so the categorical a_0 reading is -6.9% , not $+0.9\%$. v1.1 (2).
B2	BTFR $\sqrt{\ell_\tau}$ normalisation deficit ($A_\tau \approx 28.4$ vs. $A_{\text{obs}} \approx 47$) undisclosed in honest-gaps section (R3 blocker).	§7 now discloses the $\sqrt{\ell_\tau}$ deficit explicitly, flags V.T163 “open” status, and argues non-propagation to L_2 G26 observables. v1.1 (2).
B3	RMOND / AeST (Skordis-Zlosnik 2021) absent; only relativistic-MOND surviving GW170817 (R4 blocker).	§1 and §6.3 now cite [12, 13, 11] with explicit GW170817 constraint; Bekenstein TeVeS qualified as “GW170817-disfavoured”. v1.1 (3).
B4	Verlinde pairwise-kSZ claim overstated; no published cosmic-scale prediction; cluster-scale critiques absent (R4 blocker).	§4.3 and §6.3 softened: “no published pairwise-kSZ prediction”; Lelli+2017 and Hees+2017 cluster-scale critiques cited. v1.1 (3).
B5	§4.3 $f(R)$ / scalar-tensor bullet citation-free (R4 blocker).	§4.3 now cites Starobinsky, Hu-Sawicki, Horndeski, Langlois-Noui. Additional alternatives (Moffat MOG, BIMOND, modified-inertia, superfluid, fuzzy DM) added to §6.3. v1.1 (3).
B6	$\bar{\tau} \cdot v_{\text{pair}}$ optical-depth degeneracy sidestepped (R1 Tier-B).	§7 now addresses: boundary-holonomy mass is non-baryonic so f_{gas} per M_{eff} differs from Λ CDM by factor ~ 0.15 ; affects N13 amplitude (not N12 index). v1.1 (2).
B7	Condition-(iii) midpoint-gradient argument underspecified (R1 Tier-B).	§3.2 now estimates midpoint $g/a_0 \sim 10^{-4}$ at $r_{12} = 100$ Mpc, giving $ n - 2 \lesssim 10^{-10}$ parametric. v1.1 (2).
B8	Scope-of-Lean-certification block shallower than the inflation note; per-entry table missing (R2 Tier-B).	§3.7 now has a per-entry table for V.T28, V.T78, V.T97, V.T98, V.R164, V.T213, V.T215, V.T203/V.D261 with <code>TauLib</code> module paths and statuses. v1.1 (1).
B9	Fit-space / effective-rank disclosure absent; inflation note’s §4.4 analog missing (R3 Tier-B).	§4.4 added: enumerates three new structural choices (ℓ_τ scale, 6.65 saturation ratio, ν_{zch} interpolation), extends effective rank from inflation-sector 10–15 to joint 13–18; $n = 2$ costs zero additional rank. v1.1 (4).
B10	“Third way” abstract framing over-marketed; Verlinde / $f(R)$ not acknowledged as sharing the corner (R3 Tier-B, R4 Tier-B).	Abstract now states: “This corner is not exclusive to the categorical framework” with explicit Verlinde / $f(R)$ / Horndeski acknowledgment. v1.1 (4).
B11	Pre-registration antecedence language in abstract and §2 over-reads “we pre-registered” vs. “structural commitment antecedent, N-entry defined herein” (R3 Tier-B).	Abstract rewritten: “V.T28/V.T78 (recorded in Book V before the Gallardo measurement)”; N12 annotation “with the structural commitment antecedent to G26”. v1.1 (4).

#	Finding	Fix in v1.1
C1	“MOND curve pulled systematically low” awkward phrasing (R1 Tier-C).	§2 reworded to “MOND predicted amplitude falls systematically below the measured band”. v1.1 (5).
C2	D3 “null DM direct detection” unfalsifiable in strict sense (R3 Tier-C).	Appendix C D3 now conditional on experiments reaching forecasted sensitivity. v1.1 (5).
C3	Bo1teSLACS2006 in bib, never cited (R4 Tier-C).	Removed. v1.1 (5).

#	Finding	Fix in v1.2 (1)
LW ₁	V.T ₂₁₃ module path claimed BookV.Astrophysics.BulletClusterLSS; actual location BookV.Cosmology.CMBSpectrum:1175 (Buzzard blocker, V.T ₁₉₁ -analog).	§3.7 scope table corrected with explicit line number. v1.2 (1).
LW ₂	V.T ₂₁₅ module BookV.Cosmology.DarkSectorConsistency does not exist on disk; actual location BookV.Cosmology.CMBSpectrum:1240 (Buzzard blocker, fictitious module).	§3.7 corrected with line number; the fictitious module path is <i>not</i> present anywhere in v1.2. v1.2 (1).
LW ₃	V.T ₂₀₃ / V.D ₂₆₁ row listed as “—”; both are actually present at BookV.Astrophysics.RotationCurves:495 and :516 (Buzzard Tier-A).	§3.7 split into two rows with line numbers; scope column shows conjectural, formalisation column shows skeleton. v1.2 (1).
LW ₄	V.R ₁₆₄ formalized (naming-level) is a Lean source-code comment, not a declaration (Buzzard + Hossenfelder convergent).	Re-labelled as comment-only in §3.7; nomenclature-tier discussion added explicitly distinguishing comment-only from skeleton from Nat-certified from Real-certified. v1.2 (1).
LW ₅	“Formalized” used across multiple Lean strengths (Real, Nat, skeleton, comment) without distinc- tion; readers will overclaim what Lean actually checks (Buzzard Tier-B).	Four explicit nomenclature tiers introduced: Real-certified (empty for this paper), Nat-certified (V.T ₂₈ , V.T ₇₈ , V.T ₉₇ , V.T ₉₈), skeleton (V.T ₂₁₃ , V.T ₂₁₅ , V.T ₂₀₃ , V.D ₂₆₁), comment-only (V.R ₁₆₄). v1.2 (1).
LW ₆	Scope and formalisation fields conflated in v1.1 scope table; registry has them as independent fields (Buzzard Tier-A).	§3.7 table now has two separate columns: formalisation (the Lean strength tier) and scope (tau-effective / conjectural). v1.2 (1).
LW ₇	Trust-budget surface not disclosed (custom ax- ioms, Book VII methodological sorries, ~ 1,842 native_decide leaves) (Buzzard Tier-B).	Trust-budget paragraph added to §3.7, explicitly confirming that none of the paper’s Nat-certified or skeleton theorems invoke native_decide or depend on the Book VII methodological sorries, and that they do ride on the four τ -kernel custom axioms. v1.2 (1).