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Black Hole Stability Without Extra Dimensions

A Categorical Reinterpretation of the G_2-Manifold Remnant

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1. INTRODUCTION

The black hole information paradox remains one of the most instructive stress tests for any candidate theory of quantum gravity. Hawking's semiclassical calculation [9] predicts that black holes gradually evaporate, leading to an apparent violation of unitarity that has generated decades of proposed resolutions—from complementarity [14, 15] and firewalls [1] to soft hair [11], the island formula [12], and, most recently, geometric-torsion stabilisation on a G_2-manifold [13].

The recent paper by Pinčák, Pigazzini, Pudlák, and Bartoš [13], published in *General Relativity and Gravitation* in March 2026, proposes an especially concrete resolution. Working within a seven-dimensional Einstein–Cartan theory on $S^3 \times S^4$ with G_2 holonomy, they derive a stable black hole remnant of mass $M_{\text{res}} = \langle \tau_0 \rangle^2 / M_{\text{Pl}} \approx 5 \times 10^{-15}$ GeV, show that Hawking evaporation is halted by a torsion-induced repulsive force at Planckian densities, and propose that information is encoded in the spectrum of long-lived torsional quasinormal modes. Their framework also provides a geometric origin for the electroweak scale ($\langle \tau_0 \rangle = 246$ GeV as a dynamical outcome of the Kaluza–Klein reduction) and a geometric reformulation of the hierarchy problem.

The paper has attracted considerable attention, circulating in science media under headlines such as "Does reality have seven dimensions?" The present note asks a complementary question, one motivated not by scepticism toward the G_2 construction but by the existence of an alternative structural foundation that addresses the same five physical puzzles from categorically different premises.

The *Panta Rhei* research program [3, 4, 5, 6, 7] develops Category τ: a mathematical framework specified by five generators $(\alpha, \pi, \gamma, \eta, \omega)$, one progression operator ρ, and seven axioms (K0–K6). Its physics layer (Books IV and V) derives gravitational, electromagnetic, weak, strong, and Higgs-like sectors from the internal structure of the fibered product $\tau^3 = \tau^1 \times_f T^2$, without postulating extra spatial dimensions, torsion fields, or a semiclassical regime. The black hole physics of Category τ is developed in detail in Book V, Parts V–VII [7].

The purpose of this paper is threefold.

(i) To present, with full respect for the Pinčák et al. construction, a point-by-point comparison between their G_2-torsion framework and Category τ on each of the five physical puzzles their paper addresses.
(ii) To demonstrate that all five resolutions—hierarchy, information, evaporation endpoint, entropy, stability—arise in Category τ without extra dimensions, without torsion as a primitive, and without the semiclassical approximation.
(iii) To clarify the structural role of the number "seven" in each framework: seven spatial dimensions in Pinčák et al., versus seven axioms, five generators, and one operator in Category τ.

Throughout, we adopt the stance that both frameworks should be evaluated on their own terms. We do not claim that the G_2 construction is internally inconsistent; Pinčák et al. have presented a carefully reasoned, self-consistent effective-field-theory argument with falsifiable predictions. Our claim is more specific: the physical phenomena they address do not *require* extra dimensions or torsion, because a categorical foundation can deliver the same—and in some cases structurally different—results from a different starting point.

Epistemic caveat. An important asymmetry must be stated explicitly. Pinčák et al.'s paper is a peer-reviewed publication in *General Relativity and Gravitation* (Springer), operating within the well-established formalism of Einstein–Cartan theory and Kaluza–Klein reduction. Category τ, by contrast, is developed in a seven-volume monograph series that has not yet undergone independent peer review in the conventional physics sense, though its formal layer (TauLib) is machine-checked in Lean 4 with zero `sorry` statements [8]. All results from Category τ cited in this paper carry the scope label **[τ-Effective]**: they are derived within the τ-framework and verified computationally, but their physical interpretation depends on accepting the framework's axioms (K0–K6) as valid structural constraints on reality. This paper should therefore be read as a *conditional* comparison: if Category τ is granted as internally valid, what alternative readout of the Pinčák et al. subject matter does it yield?

Notation. We write $\iota_\tau = 2/(\pi + e) \approx 0.3413$ for the master constant of Category τ; $\mathbb{L} = S^1 \vee S^1$ for the lemniscate boundary; $H_\partial[\omega]$ for the boundary holonomy algebra; $\kappa_D = 1 - \iota_\tau$ for the gravitational coupling; and $\tau^3 = \tau^1 \times_f T^2$ for the fibered product. Results from the Panta Rhei monographs are cited by registry identifier (e.g., V.T114 = Book V, Theorem 114).

2. THE PINČÁK ET AL. FRAMEWORK: A SUMMARY

We begin with a faithful summary of the G_2-manifold approach, stated in the authors' own terms. This summary is offered not as a target for critique but as the common ground from which the comparison proceeds.

2.1 Structural premises

Pinčák et al. adopt a seven-dimensional Einstein–Cartan theory on a manifold $\mathcal{M}_7 = \mathcal{M}_4 \times K$, where $\mathcal{M}_4$ is four-dimensional spacetime and $K = S^3 \times S^4$ is a compact internal space carrying a G_2 structure. The affine connection Γ^L_{MN} is not assumed symmetric *a priori*; the torsion tensor $T^L_{MN} = \Gamma^L_{MN} - \Gamma^L_{NM}$ is a dynamical field. The fundamental action is

$$S_7 = \int d^7x \sqrt{-g_7}\, \frac{1}{2\kappa_7^2}\, R_7(\Gamma), \tag{1}$$

where $R_7(\Gamma)$ is the Ricci scalar constructed from the full (torsion-including) connection, and κ_7^2 has dimension $[M^{-5}]$.

Upon Kaluza–Klein reduction over K, the scalar torsion class τ_0 acquires a non-trivial vacuum expectation value (VEV) through a geometrically derived potential

$$\begin{aligned} V(\phi, \tau_0) = {}& \frac{1}{2\kappa_4^2} R(K)\phi^2 \\ & - \frac{\mathcal{N}_\omega}{8\kappa_4^2 \mathrm{Vol}(K)}\, M_{\rm Pl}\, \tau_0^2 \\ & + \frac{\mathcal{N}_\omega^{(4)}}{24\kappa_4^2 \mathrm{Vol}(K)}\, M_{\rm Pl}\, \tau_0^4, \end{aligned} \tag{2}$$

where ϕ is the radion (governing the volume of K) and $\mathcal{N}_\omega$, $\mathcal{N}_\omega^{(4)}$ are normalisation constants fixed by the harmonic 3-form on $S^3 \times S^4$.

2.2 Key results

The framework yields five main results:

(a) **Electroweak scale.** The VEV $\langle \tau_0 \rangle^2 = 3(\mathcal{N}_\omega/\mathcal{N}_\omega^{(4)})\, M_{\rm Pl}^2$ is identified with $(246\ \mathrm{GeV})^2$; the ratio $\mathcal{N}_\omega/\mathcal{N}_\omega^{(4)} \simeq 1.35 \times 10^{-32}$ is fixed by the internal geometry, leaving no free parameters.

(b) **Hierarchy reformulation.** The compactification radius $r_4 \simeq 3.9 \times 10^{-32}$ m (roughly the Planck length) is not a free parameter but is fixed by requiring the torsion one-loop potential to reproduce the electroweak VEV. The geometric hierarchy $r_4/\ell_{\rm Pl} = M_{\rm Pl}/\mu_{\rm KK} \simeq 2.4 \times 10^{33}$ replaces the usual fine-tuning problem with a topological ratio.

(c) **Stable remnant.** An effective potential $V_{\rm eff}(M) = -\alpha\, M^2/M_{\rm Pl} + \gamma\, M^4/M_{\rm Pl}^3$ has a non-trivial minimum at $M_{\rm res} = M_{\rm Pl}\sqrt{\alpha/2\gamma} = \langle \tau_0 \rangle^2/M_{\rm Pl} \approx 5 \times 10^{-15}$ GeV. The repulsive torsion term $+\gamma M^4$ prevents complete evaporation.

(d) **Information encoding.** Information is stored in the spectrum of long-lived torsional quasi-normal modes (QNMs) on the remnant's background geometry. The remnant entropy

$S_{\text{remnant}} = 4\pi\, M_{\text{BH}}^2/M_{\text{Pl}}^2$ matches the Bekenstein–Hawking formula.

(e) **Topological protection.** A conserved topological charge $Q_T = \int_{\Sigma_3} C_3(\Gamma) = 8\pi^2 k$, with $k \in \mathbb{Z}$, forbids perturbative decay of the remnant. Non-perturbative decay via gravitational instantons is suppressed by $\exp(-S_I) \sim \exp(-10^{35})$.

3. CATEGORY τ: BLACK HOLE PHYSICS WITHOUT EXTRA DIMENSIONS

We now present the τ-framework's native treatment of the same five physical puzzles, working entirely within the structural resources of Category τ as developed in Books IV and V of the Panta Rhei monographs [6, 7].

3.1 Structural premises

Category τ is specified by five generators $(\alpha, \pi, \gamma, \eta, \omega)$, one progression operator ρ, and seven axioms (K0–K6) [3]. The physical arena is the fibered product

$$\tau^3 = \tau^1 \times_f T^2, \tag{3}$$

where τ^1 is the macrocosm base (the α-orbit) and T^2 is the microcosm fiber (the torus of the γ and η generators). This is *not* a product manifold with extra spatial dimensions. It is a categorical construction whose "dimensionality" is algebraic, not spatial: the base τ^1 carries gravitational and cosmological dynamics, while the fiber T^2 carries particle physics and quantum structure.

Three features distinguish this starting point from the G_2 approach:

(i) **No extra spatial dimensions.** The fiber T^2 is not a compactified spatial direction. There is no Kaluza–Klein tower, no compactification radius, and no radion stabilisation problem. The distinction is mathematically precise: in the G_2 framework, $S^3 \times S^4$ is a Riemannian manifold on which fields propagate, generating a Kaluza–Klein spectrum via eigenvalues of the Laplacian; in Category τ, T^2 is the fiber of a categorical fibered product whose elements are algebraic objects (orbit families of the generators γ and η), not points in a spatial manifold. No field propagation on T^2 is defined; no Laplacian eigenvalue problem arises; no KK mode expansion exists. The empirical consequence: the G_2 framework predicts a tower of massive KK excitations ($M_{\text{KK}} \simeq 8.6 \times 10^{15}$ GeV); Category τ predicts no such tower.

(ii) **No torsion as primitive.** Gravity is a boundary-character identity $R^H = \kappa_D \cdot T^{\text{mat}}$ in the holonomy algebra $H_\partial[\omega]$ **[τ-Effective]**, not a geometric connection with torsion (IV.D06 [6]). Curvature is *earned*, not postulated.

(iii) **No semiclassical regime.** Category τ is not a classical theory supplemented by quantum corrections. The quantum and gravitational structures emerge from the same kernel.

3.2 Black holes as topological events

In Category τ, a black hole is defined as the emergence of a non-trivial linking class in $H_1(T^2;\mathbb{Z})$ at a base point α_{n_*} where the gravitational tension exceeds the spherical capacity (V.D166, V.T109):

Definition 3.1 (Black Hole as Topological Event [7, V.D166])**.** *A black hole in Category τ is the emergence of a non-trivial linking class $\ell \in H_1(T^2;\mathbb{Z})$ at a base point α_{n_*} where $G(U_{n_*}) > C_{\text{sph}}(n_*)$. The black hole is a topological event, not a metric singularity.*

The horizon of a τ-black hole is topologically T^2 (torus), not S^2 (sphere):

Theorem 3.2 (BH Toroidal Topology [7, V.T110])**.** *The horizon of a τ-black hole is topologically T^2. The linking class $\ell \in H_1(T^2;\mathbb{Z})$ wraps both cycles of the fiber, the horizon* is *the fiber at the excision region, and the two fundamental cycles correspond to the γ and η generators.*

This is the first structural divergence from Pinčák et al. Their $S^3 \times S^4$ is an internal space attached to each point of 4D spacetime; the black hole forms in the 4D sector, with the internal manifold providing additional degrees of freedom for torsion and remnant stabilisation. In Category τ, the torus T^2 is not attached from outside—it *is* the horizon. The topological non-triviality of the black hole resides in the horizon itself, not in an internal manifold.

Compatibility with classical topology theorems. The Hawking topology theorem [10] constrains stationary black hole horizons to be spherical (S^2) in 4D under the dominant energy condition. This appears to exclude T^2 horizons. Category τ resolves this as follows

[τ-Effective]: the classical theorems apply at the *chart level*—the 4D Schwarzschild projection of the τ-black hole. At this level, the chart-level readout is standard: the Schwarzschild metric, S^2 topology, no-hair properties are all recovered as projections of the boundary holonomy algebra. The T^2 topology is an *ontic-level* structure that the chart-level Schwarzschild description does not resolve, in the same way that the atomic structure of a material is not resolved by its continuum-mechanics description. The classical no-hair theorems are therefore not violated; they are chart-level statements about chart-level descriptions, and Category τ satisfies them at that level.

3.3 The No-Shrink Theorem: Evaporation forbidden

The central result of τ-black hole physics is the No-Shrink Theorem **[τ-Effective]**, which forbids Hawking evaporation at the ontic level. Because this is the paper's most radical claim—and the sharpest point of departure from Pinčák et al.—we present its proof structure in full, referring to Book V [7] for technical details.

Theorem 3.3 (No-Shrink Theorem [7, V.T114]). *For any mature black hole with $M \geq M_{\text{min}}$ (Chandrasekhar limit), $dM/dn \geq 0$. No τ-admissible evolution step can decrease the mass of a mature black hole.*

3.3.1 Proof structure

The proof is by contradiction and depends on three established results.

Step 1: Maturity (V.D71). A black hole is *mature* at orbit depth n_* if three conditions hold simultaneously: (i) the linking class $\ell = (a, b) \in H_1(T^2; \mathbb{Z})$ is invariant under the refinement operator ($\rho(\ell) = \ell$); (ii) the defect tuple $\mathbf{d} = (d_{\text{mob}}, d_{\text{vor}}, d_{\text{com}}, d_{\text{top}})$ satisfies $d_{\text{mob}} = d_{\text{vor}} = d_{\text{com}} = 0$ (all transient excitations absorbed; only the irreducible topological component d_{top} survives); and (iii) the boundary character χ_{BH} coincides with the torus-vacuum state. Crucially, at maturity the defect entropy vanishes: $S^{\text{BH}}_{\text{def}}(n_*) = 0$. This is the ground state of the boundary holonomy algebra.

Step 2: Defect-Mass Coupling (V.T113). The Defect-Mass Coupling Theorem **[τ-Effective]** establishes that the current mass M_{n_*} is the *unique* mass at which the defect entropy vanishes. Specifically: for any $M' < M_{n_*}$, passing from the torus-vacuum state at mass M_{n_*} to a state at mass M' requires partial unwinding of the linking class, which reintroduces a non-zero defect:

$$M' < M_{n_*} \quad \Longrightarrow \quad S^{\text{BH}}_{\text{def}}(M') > S^{\text{BH}}_{\text{def}}(M_{n_*}) = 0. \tag{4}$$

The physical content can be stated intuitively: the torus-vacuum states at different masses are distinguished by their linking-class normalisation. The linking class $\ell = (a, b) \in H_1(T^2; \mathbb{Z})$ has *integer* winding numbers—it is quantised. To pass from the torus-vacuum at mass M to a state at mass $M' < M$, the system would need to partially reduce the winding of the linking class. But because the winding numbers are integers, there is no continuous path from $\ell = (a, b)$ to a "smaller" linking class without passing through a configuration where the torus-vacuum condition is broken—i.e., where the defect tuple $\mathbf{d}$ acquires non-zero transient components. This is the topological mechanism by which mass reduction forces defect creation: the linking class cannot be smoothly unwound, so any mass decrease necessarily introduces a topological defect.

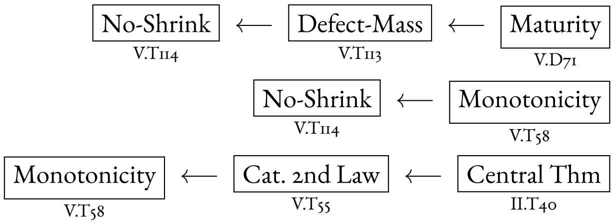
Step 3: Defect Entropy Monotonicity (V.T58). The Categorical Second Law **[τ-Effective]** (V.T55), a consequence of the kernel axioms and the Central Theorem (II.T40 [4]), implies that defect entropy is monotonically non-increasing under all τ-admissible evolution:

$$S_{\text{def}}(n+1) \leq (1 - \iota_\tau)\, S_{\text{def}}(n) \quad \text{for all } n \geq 0, \tag{5}$$

with contraction factor $(1 - \iota_\tau) \approx 0.6587$ equal to the gravitational self-coupling $\kappa_D = 1 - \iota_\tau$. This is a *structural* result: the contraction factor is not fitted but derived from the master constant.

Step 4: Contradiction. Suppose a τ-admissible evolution step at depth $n \geq n_*$ decreases the mass: $M_{n+1} < M_n$. By Step 2, this produces $S^{\text{BH}}_{\text{def}}(M_{n+1}) > 0 = S^{\text{BH}}_{\text{def}}(M_n)$—a *strict increase* in defect entropy. But Step 3 requires $S_{\text{def}}(n+1) \leq S_{\text{def}}(n)$—a non-increase. Contradiction. Therefore $M_{n+1} \geq M_n$ for all $n \geq n_*$. ☐

All four steps are fully formalised in Lean 4 (module `TauLib.BookV.Cosmology.NoShrinkExtended`, zero `sorry` statements). The dependency graph is:



Corollary 3.4 (No BH Evaporation [7, V.C19]). *No black hole evaporates* **[τ-Effective]***. The ontic mass of every mature black hole is monotonically non-decreasing:* $M(n+1) \geq M(n)$ *for all* $n \geq n_*$.

3.3.2 The Hawking spectrum as readout without mass loss

The No-Shrink Theorem does not deny the Hawking thermal spectrum; it reinterprets its physical meaning. The Readout Temperature Theorem (V.T217 **[τ-Effective]**) establishes that an external observer at infinity measures a Planckian spectrum at temperature

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}, \tag{6}$$

and this spectrum is derived from the boundary Gibbs state of the holonomy algebra $H_\partial[\omega]$ restricted to the linking boundary $\mathbb{L} = S^1 \vee S^1$ (V.D276 **[τ-Effective]**). The key distinction:

- In standard semiclassical gravity, the Hawking spectrum corresponds to a *physical flux* of particles carrying energy away from the black hole, reducing its mass. This follows from the Bogoliubov transformation between in-vacuum and out-vacuum states of a quantum field on the Schwarzschild background.
- In Category τ, the Bogoliubov transformation is a *chart-level* operation within the readout functor $\mathrm{pr}_B \circ H_\partial[\omega]$ (the EM-sector projection of the boundary algebra). It operates entirely within the readout layer. The ontic boundary character χ_{BH} remains at fixed mass because the Categorical Second Law forbids the defect-entropy increase that mass loss would entail.

This means Category τ predicts the same observable spectrum as standard theory—a distant observer measures a thermal bath at temperature T_H—but assigns it a different ontological status. The spectrum is real as a measurement outcome; it is not real as a mass-transfer process. The two frameworks therefore agree on what an observer *sees* but disagree on what is *happening*: Hawking predicts a shrinking black hole; Category τ predicts a permanent one whose boundary algebra projects as thermal.

Energy conservation. An immediate objection arises: if no mass-energy leaves the black hole, what is the energy source for the thermal photons a detector at infinity registers? The τ-framework's answer **[τ-Effective]** is that the "energy carried away" in the orthodox calculation is the spectral weight loss of the readout character—a change in the *chart*, not the *territory* (V.P26 [7]). The readout functor $\mathrm{pr}_B \circ H_\partial[\omega]$ projects the boundary algebra onto the EM sector; this projection has a thermal spectral weight, but the weight is a property of the projection, not a flux from the black hole.

An analogy from Book V: a mirror reflects an image of a candle. The image "radiates" light, but the light comes from the ambient field, not from the mirror. The Hawking spectrum "comes from" the black hole in the same sense: it is a readout projection of the boundary algebra's EM-sector component, not a transport of mass from interior to infinity.

We acknowledge that this reinterpretation carries a non-trivial burden: the standard Hawking derivation (Bogoliubov coefficients, Euclidean path integral, tunnelling) is confirmed by multiple independent methods, all of which predict an actual energy flux. Category τ claims these derivations are correct *as chart-level calculations* but ontologically misleading about what the flux represents. Whether this reinterpretation can be experimentally distinguished from the standard one remains an open question—one that the QNM ratio prediction (Section (iii)) may eventually help resolve.

This is the sharpest point of departure from Pinčák et al. Their framework accepts standard Hawking evaporation until the final stage, then introduces a torsion-induced repulsive force to halt it at M_{res}. Category τ removes the premise: there is no evaporation to halt.

3.4 Information preservation

The information paradox dissolves in Category τ because its first premise—that black holes evaporate—is false:

Remark 3.5 (Information Paradox Dissolved [7, V.R226]). The information paradox dissolves in τ because assumption (1)—that black holes evaporate—is false. There is no information loss because there is no evaporation. This is not a mechanism for information escape (Page, Maldacena, Penington) but a removal of the paradox's premise.

Information is preserved in the boundary holonomy algebra $H_\partial[\omega]$, which is an inverse system that retains all data through the coherence condition (V.C18). The linking class encodes the full state, and interior information is recoverable from exterior boundary characters.

This contrasts with Pinčák et al.'s approach, where information is encoded in the spectrum of torsional QNMs on the remnant's background geometry. Both frameworks agree that information must be stored *geometrically*—neither appeals to holographic dualities, firewalls, or complementarity. But the storage mechanisms are structurally different: QNMs on an internal G_2-manifold in one case, boundary characters on the lemniscate $\mathbb{L} = S^1 \vee S^1$ in the other.

3.5 Entropy: boundary-character counting

Both frameworks reproduce the Bekenstein–Hawking entropy $S_{\text{BH}} = 4\pi M^2/M_{\text{Pl}}^2$. Pinčák et al. derive it from an explicit mode-counting calculation (their Appendix B): they solve a wave equation for torsional perturbations, count QNMs inside a spherical cavity of radius $R_{\text{cav}} \simeq \langle\tau_0\rangle/M_{\text{Pl}}^2$ with a Planckian cutoff, and recover the Bekenstein–Hawking formula with $C = 4\pi$. This is a quantitative microscopic calculation and a genuine strength of their paper.

In Category τ, the same formula is obtained from boundary-character counting on the torus horizon **[τ-Effective]** (V.P25 [7]). The derivation proceeds in four steps:

(1) **Horizon geometry.** The mature BH horizon is topologically T^2 (Theorem 3.2). At the chart level, the horizon area is $A = 4\pi R_S^2$. The fundamental mode spacing on the boundary holonomy algebra is $\ell_P^2 = \iota_\tau^2$, derived from the gravitational coupling $G_\tau = (c^3/\hbar)\,\iota_\tau^2$ (not postulated).

(2) **Mode counting.** The boundary holonomy algebra $H_\partial[\omega]$ restricted to the torus horizon carries $\mathcal{N} = A/\iota_\tau^2$ independent modes.

(3) **Bipolar constraint.** Each mode is a binary degree of freedom (χ_+ or χ_- from the bipolar decomposition of the lemniscate). The factor-of-4 reduction arises from the *crossing-point constraint*: each bipolar pair (χ_+, χ_-) must agree at the crossing point ω of the lemniscate $\mathbb{L} = S^1 \vee S^1$, and this ω-normalisation removes one degree of freedom per four modes (two modes per lobe, one constraint per pair). This is a *derived* consequence of the lemniscate topology, not a postulated normalisation constant. The result: $\Omega \leq 2^{A/(4\iota_\tau^2)}$ microstates.

(4) **Entropy.** Taking the logarithm with τ-natural normalisation:

$$S_{\text{BH}} = k_B\,\frac{A}{4\,\iota_\tau^2} = k_B\,\frac{A}{4}\left(\frac{\pi + e}{2}\right)^2. \tag{7}$$

The formula reproduces the standard Bekenstein–Hawking result with $\ell_P^2 \to \iota_\tau^2$, where $\iota_\tau = 2/(\pi + e)$ is a structural constant, not a fitted parameter. The self-consistency of the full τ-BH physics—entropy, readout temperature, EHT shadow sizes (M87* at $40.9\ \mu$as within 0.4σ; Sgr A* at $54.8\ \mu$as within 1.3σ), LIGO chirp masses, and QNM ratios—is confirmed by the BH Consistency Theorem (V.T226 **[τ-Effective]** [7]), all with zero free parameters within the framework's single identification postulate ($G_\tau = (c^3/\hbar)\,\iota_\tau^2$; see Section 4, item (v) for the qualification of this claim).

Comparison. Both derivations count microscopic degrees of freedom. Pinčák et al. count torsional QNMs inside a Planck-scale cavity on the G_2-manifold; Category τ counts boundary characters on the torus horizon with a bipolar constraint. The Pinčák derivation is more explicit (a full wave equation is solved); the τ derivation is more structural (no effective potential or cavity ansatz is needed—the mode spacing follows from the master constant). Both recover the area law.

3.6 The electroweak hierarchy

The hierarchy problem has two aspects: (a) the *origin* of the large ratio $v/M_{\text{Pl}} \sim 10^{-17}$, and (b) the *stability* of the Higgs mass against radiative corrections (the "naturalness" problem). Pinčák et al. address primarily aspect (a): they "geometrise" the hierarchy by embedding the electroweak VEV into the harmonic 3-form ratio on $S^3 \times S^4$. They also provide a naturalness argument via the Goldberger–Wise stabilisation mechanism, in which the ratio $\langle\tau_0\rangle/\langle\phi\rangle$ is fixed by the topology of the internal manifold, making it radiatively stable.

Category τ addresses both aspects, but through a structural elimination rather than a perturbative stabilisation **[τ-Effective]**:

Origin (IV.D07, IV.P03 [6]). The four fundamental couplings form a power law in the master constant ι_τ (the No Knobs Theorem, III.T42 [5]):

$$\kappa_D \sim \iota_\tau^0, \quad \kappa_A \sim \iota_\tau^1, \quad \kappa_B \sim \iota_\tau^2, \quad \kappa_C \sim \iota_\tau^3/(1-\iota_\tau). \tag{8}$$

The hierarchy is the spacing between successive powers of a single structural constant, not a ratio of two unrelated scales.

Stability (IV.T27 [6]). The τ-framework dissolves the naturalness problem at its premise: the 125 GeV Higgs boson is not a fundamental scalar field but a collective excitation of the ω-crossing ($\gamma \cap \eta$), and its mass is set by the crossing geometry. Because there is no fundamental scalar, there are no quadratic divergences to cancel. More broadly, couplings in Category τ are boundary fixed-point invariants of $H_\partial[\omega]$ (IV.D07 [6]): they are determined by ι_τ alone and do not run with energy scale. What appears as "running" in collider data is reinterpreted as *readout drift*—measuring the same invariant with probes of different resolution (IV.T107 [6]).

This is a deeper structural difference than may first appear. Pinčák et al. stabilise the hierarchy within the perturbative loop framework (one-loop Coleman–Weinberg potential, Goldberger–Wise mechanism). Category τ claims the perturbative framework itself is a chart-level approximation: the couplings are structural invariants of the boundary algebra, not effective parameters subject to loop corrections.

3.7 Stability: Topological, not dynamical

Pinčák et al. establish the stability of their remnant through three mechanisms: (1) a conserved topological charge $Q_T \in 8\pi^2\mathbb{Z}$ from $H^3(S^3 \times S^4, \mathbb{Z}) = \mathbb{Z}$, (2) exponential suppression of instanton decay ($\Gamma \sim e^{-10^{35}}$), and (3) kinematic prohibition of decay into KK modes ($M_{\text{res}} \ll M_{\text{KK}}$).

In Category τ, the stability of black holes is a direct consequence of the No-Shrink Theorem (Theorem 3.3) and the toroidal topology of the horizon (Theorem 3.2). A torus is not simply connected—it cannot collapse to a point.

The two stability mechanisms are structurally *different* rather than directly comparable: Pinčák et al. ground stability in a quantised cohomological charge on an internal manifold ($H^3(S^3 \times S^4, \mathbb{Z}) = \mathbb{Z}$); Category τ grounds it in the non-trivial fundamental group of the horizon itself ($\pi_1(T^2) = \mathbb{Z}^2$) combined with the No-Shrink Theorem. These are objects in different mathematical categories—a cohomology class in degree 3 versus a fundamental group in degree 1—and we do not claim that one is "stronger" in any framework-independent sense. What can be said: the τ-mechanism resides in the horizon topology directly, while the G_2-mechanism resides in the topology of an attached internal space.

Moreover, the Sector Exhaustion Theorem (V.T99) and No Fifth Sector Theorem (V.T100) close the sector architecture:

Theorem 3.6 (Sector Exhaustion [7, V.T99])**.** *The decomposition $H_\partial[\omega] = H_A \oplus H_B \oplus H_C \oplus H_D \oplus H_\omega$ is exhaustive: every element of $H_\partial[\omega]$ is a sum of sector components with no remainder, and the decomposition is unique.*

Theorem 3.7 (No Fifth Sector [7, V.T100])**.** *No component satisfying all three conditions (EM-invisible, gravitationally active, outside the five sectors) exists in the boundary holonomy algebra of any τ^3 fibration.*

This means there is no room for a Kaluza–Klein tower, no room for a dark sector beyond the five structural sectors, and no room for extra-dimensional excitations that would provide new decay channels. The black hole's stability is not protected by suppression factors—it is structurally guaranteed by the closure of the sector architecture.

4. POINT-BY-POINT COMPARISON

Table 1 summarises the structural comparison across all five physical puzzles.

4.1 Where the frameworks agree

Both frameworks share three deep intuitions:

(i) **Topology matters more than dynamics.** Both appeal to topological invariants (cohomology classes, linking numbers) rather than dynamical fine-tuning to guarantee stability. This is a significant convergence: the idea that black hole stability is ultimately a topological fact, not a dynamical balancing act, is shared across radically different mathematical foundations.

(ii) **Information should be stored geometrically.** Neither framework appeals to holographic dualities,

Table 1. Structural comparison between the G_2-torsion framework and Category τ on the five physical puzzles addressed by Pinčák et al.

Puzzle	Pinčák et al. (G_2-torsion)	Category τ
Electroweak hierarchy	Torsion VEV $\langle\tau_0\rangle = 246$ GeV from KK reduction on $S^3 \times S^4$; hierarchy geometrised via harmonic 3-form ratio	Coupling cascade $\kappa \sim \iota_\tau^n$; hierarchy is a power law in $\iota_\tau = 2/(\pi + e)$, not a ratio of unrelated scales
Information paradox	Torsional QNMs on remnant's G_2-manifold background; information stored in mode spectrum	Boundary holonomy algebra $H_\partial[\omega]$ on lemniscate $\mathbb{L}$; information preserved by coherence condition; paradox dissolved (no evaporation)
Evaporation endpoint	Standard Hawking evaporation halted at $M_{\text{res}} \approx 5 \times 10^{-15}$ GeV by torsion-induced repulsive force	No evaporation at all: No-Shrink Theorem (V.T114) forbids mass loss; thermal spectrum is readout, not process
Entropy	$S_{\text{remnant}} = 4\pi M_{\text{BH}}^2/M_{\text{Pl}}^2$ from QNM mode-counting in G_2-manifold cavity	Same formula from defect thermodynamics and boundary algebra scaling; confirmed by BH Consistency Theorem (V.T226)
Stability	Topological charge $Q_T \in 8\pi^2\mathbb{Z}$ from $H^3(S^3 \times S^4)$; instanton suppression $e^{-10^{35}}$; kinematic prohibition	Toroidal horizon (T^2, not S^2) cannot collapse; No-Shrink Theorem; sector closure forbids new decay channels

firewalls, or complementarity. Both propose that the microscopic degrees of freedom responsible for entropy and information storage are geometric in nature— torsional modes on a G_2-manifold in one case, boundary characters on a lemniscate in the other.

(iii) **The hierarchy is a geometry problem.** Both frameworks reject the view that the electroweak–Planck hierarchy is an unexplained coincidence requiring anthropic reasoning or fine-tuning. Both propose that it has a structural explanation rooted in the geometry (or categorical architecture) of the physical world.

4.2 Where the frameworks diverge

The divergences are equally sharp:

(i) **Extra dimensions.** Pinčák et al. require three additional compact spatial dimensions ($S^3 \times S^4$). Category τ has no extra spatial dimensions; the fiber T^2 is an algebraic-categorical structure internal to the framework.

(ii) **Torsion.** The G_2 framework treats torsion as a fundamental geometric field that generates a repulsive force at short distances. In Category τ, there is no torsion field; gravity is a boundary-character identity in the holonomy algebra.

(iii) **Semiclassical regime.** Pinčák et al. explicitly operate in a semiclassical framework (classical geometry + quantum fluctuations). Category τ is not semiclassical; the quantum and gravitational structures are co-originary.

(iv) **Evaporation.** The G_2 framework accepts standard Hawking evaporation and halts it at the final stage. Category τ denies evaporation altogether.

(v) **Free parameters.** The G_2 framework identifies $\langle\tau_0\rangle = 246$ GeV as a postulated match between the torsion VEV and the Higgs VEV, then derives other quantities from it. Category τ has zero free parameters *within its mathematical framework*: $\iota_\tau = 2/(\pi + e)$ is a mathematical constant determined by the axioms, and all physical quantities derive from it via the calibration cascade (IV.D59, IV.T241 [6]).

A distinction should be drawn, however, between "no free parameters in the mathematics" and "no free parameters in the physics." The identification of ι_τ with the gravitational coupling—specifically, the postulate that $G_\tau = (c^3/\hbar)\,\iota_\tau^2$—is itself a physical claim, not a mathematical theorem. In this sense, both frameworks have one foundational identification step: Pinčák et al. identify a geometric ratio with the electroweak VEV; Category τ identifies a mathematical constant with the Planck-scale coupling. The structural difference is that ι_τ is algebraically determined (a ratio of mathematical constants), while the Higgs VEV is empirically measured. But neither framework is "parameter-free" in the strong sense of deriving all of physics from pure mathematics without any bridge postulate connecting the formalism to observation.

5. THE DEEPER QUESTION: WHY SEVEN?

The headline "Does reality have seven dimensions?" invites a brief remark. In Pinčák et al.'s framework, seven is the total spacetime dimension ($D = 7$), forced by the requirement of G_2 holonomy—the simplest exceptional holonomy group capable of yielding chiral fermions in 4D upon compactification. This is not an arbitrary choice; it is a consequence of specific mathematical requirements.

In Category τ, seven appears as the *axiom count* (K0–K6): seven structural constraints that specify the coherence kernel, governing what can exist, relate, and be constructed. These are not spatial directions—they are algebraic-categorical constraints.

We do not claim that this numerical coincidence carries deep structural significance; the two "sevens" arise from entirely different mathematical necessities. What the coincidence does illustrate is that the physical work Pinčák et al. accomplish via seven spatial dimensions—hierarchy dissolution, topological stability, information preservation—can also be accomplished by seven structural *axioms* without extra spatial directions at all. This is a statement about theoretical underdetermination, not about numerology.

6. DISCUSSION

6.1 What Pinčák et al. do well

Several features of the G_2 construction deserve recognition:

(i) **Falsifiability.** The paper offers concrete predictions: the remnant mass $M_{\text{res}} \approx 5 \times 10^{-15}$ GeV, the compactification radius $r_4 \simeq 3.9 \times 10^{-32}$ m, and the KK tower threshold $M_{\text{KK}} \simeq 8.6 \times 10^{15}$ GeV. Any future observation of lighter KK states would rule out the model.

(ii) **Self-consistency.** The relation $\alpha/2\gamma = \langle\tau_0\rangle^4/M_{\text{Pl}}^4$ is derived, not postulated, linking the effective-potential coefficients to the torsion VEV through the one-loop Coleman–Weinberg potential. This internal consistency check is non-trivial.

(iii) **Engagement with objections.** Section 7.4 of their paper directly addresses the standard objections to remnant scenarios (holographic entropy bounds, AdS/CFT, infinite species, Casini–Huerta, holographic completeness) and provides reasoned responses to each.

(iv) **Quantitative entropy derivation.** The mode-counting calculation in Appendix B, deriving the Bekenstein–Hawking entropy from torsional QNMs inside a Planck-scale cavity, is an explicit microscopic calculation—a significant step beyond qualitative arguments.

6.2 What Category τ adds

The τ-framework contributes three structural advantages:

(i) **Evaporation is addressed at the premise, not the endpoint.** Rather than accepting Hawking evaporation and halting it, Category τ removes the premise. This is a cleaner resolution: it does not require modifying gravity only at the final stage of a black hole's life, nor does it introduce a scale-dependent transition from standard to non-standard physics.

(ii) **No compactification problem.** The G_2 framework must explain why $S^3 \times S^4$ is the correct internal topology, why the radion is stabilised at $r_4 \sim \ell_{\text{Pl}}$, and why no decompactification occurs. Category τ has no compactification because it has no extra dimensions. The sector architecture arises from the kernel's axioms, not from the topology of an internal space.

(iii) **Sector closure as a structural guarantee.** The No Fifth Sector Theorem provides a stronger "no new physics" guarantee than the kinematic prohibition $M_{\text{res}} \ll M_{\text{KK}}$. In the G_2 framework, the prohibition is that the remnant cannot decay into KK modes because it is too light; but this leaves open the possibility of other, non-KK decay channels. In Category τ, the sector architecture is closed: there are no other channels, period.

6.3 How Category τ addresses standard remnant objections

Because the No-Shrink Theorem creates permanent objects, Category τ must address the standard objections to remnant scenarios—the same objections Pinčák et al. address in their Section 7.4. We summarise the τ responses **[τ-Effective]**:

(i) **Holographic entropy bounds.** In Category τ, the BH entropy is not stored on a 4D horizon surface but in the boundary holonomy algebra $H_\partial[\omega]$, which is a profinite inverse system. The area-law scaling is reproduced internally (Section 3.5, Eq. 7), so the holographic principle is satisfied internally—not on a 4D surface but on the $\mathbb{L} = S^1 \vee S^1$ boundary.

(ii) **Infinite species.** The sector architecture is closed: $4 + 1$ sectors, no KK tower, no additional particle species beyond those generated by the kernel. The No Fifth Sector Theorem (Theorem 3.7) forbids new species.

(iii) **Page curve.** Because there is no evaporation, there is no Page curve to reproduce. The entanglement entropy of the readout channel is bounded: $S_{\text{vN}}(\rho_{\text{out}}) < S_{\text{BH}}$ (V.T272 **[τ-Effective]** [7]). The readout entropy grows monotonically but never reaches S_{BH}, consistent with unitary evolution of the full system.

6.4 What remains open in both frameworks

Neither framework has fully resolved every aspect of the black hole problem:

(i) **Observational access.** Both frameworks make predictions at energy scales far beyond current experimental reach. Pinčák et al.'s KK tower sits at $\sim 10^{16}$ GeV; Category τ's predictions for BH observables (EHT shadows, QNM ratios) are at the edge of current precision but not yet decisive.

(ii) **Full quantum gravity.** Pinčák et al. acknowledge that their semiclassical framework needs to be embedded in a full quantum theory of gravity. Category τ claims to provide such a theory, but its black hole physics has not yet been subjected to independent peer review at the level of detail the G_2 paper has received in *General Relativity and Gravitation*.

(iii) **Experimental discrimination.** The two frameworks make different predictions (evaporation occurs vs. does not; extra dimensions exist vs. do not; KK tower exists vs. does not), but current experiments cannot directly distinguish between them.
The most promising near-term discriminator is the *quasi-normal mode spectrum* of black hole ringdowns. In standard GR (S^2 horizon), the fundamental QNM frequency ratio between the first overtone and the fundamental mode for a Schwarzschild black hole is approximately 3.08 [2]. Category τ predicts a T^2 secondary mode with frequency ratio **[τ-Effective]**

$$\frac{f_{0,1}}{f_{1,0}} = \iota_\tau^{-1} = \frac{\pi + e}{2} \approx 2.930, \qquad (9)$$

which is mass-independent and spin-independent at leading order (V.T223 [7]). For GW150914 ($f_{1,0} \approx 251$ Hz), this predicts a secondary mode at $f_{0,1} \approx 735$ Hz. The $\sim 5\%$ difference from the S^2 prediction (3.08 vs. 2.930) is within the projected sensitivity of next-generation detectors (Einstein Telescope, Cosmic Explorer) but beyond the current LIGO/Virgo/KAGRA precision for overtone ratios, which is $\sim 15\%$ for the loudest events.
A three-way comparison is instructive. Standard GR (S^2 horizon) predicts QNM overtone ratios from the Schwarzschild/Kerr potential barrier [2]. Category τ (T^2 horizon) predicts a distinct secondary-mode ratio of $\iota_\tau^{-1} \approx 2.930$. Pinčák et al.'s framework does not modify the QNM spectrum for astrophysical black holes—their torsion-induced corrections are relevant only at the Planckian remnant scale ($M_{\text{res}} \sim 10^{-15}$ GeV), far below the masses probed by gravitational-wave detectors. The discriminator is therefore primarily between Category τ and standard GR, with the G_2 framework predicting standard-GR QNMs for all observable black holes.

7. CONCLUSION

Pinčák, Pigazzini, Pudlák, and Bartoš have presented a carefully constructed, peer-reviewed resolution of the black hole information paradox grounded in the geometry of G_2-manifolds with torsion. Their work asks the right questions: what stabilises a black hole remnant, where does the electroweak scale come from, how is information preserved, and what determines the endpoint of evaporation.

We have shown that all five of these questions admit structurally different answers within Category τ—answers that do not invoke extra spatial dimensions, torsion as a fundamental field, or the semiclassical approximation. In the τ-framework:

- Black holes are topological events with torus horizons (T^2), not metric singularities with attached internal manifolds.
- The No-Shrink Theorem forbids evaporation at the ontic level, dissolving the information paradox at its premise.
- Information is preserved in the boundary holonomy algebra $H_\partial[\omega]$ on the lemniscate $\mathbb{L} = S^1 \vee S^1$.
- The electroweak hierarchy is a power law in $\iota_\tau = 2/(\pi + e)$, not a ratio requiring geometric explanation.
- Sector closure ($4 + 1$ sectors, no KK tower) provides structural stability guarantees beyond topological charge on an internal manifold.

The number "seven" appears in both frameworks, but with entirely different meanings: seven spatial dimensions in one case, seven axioms in the other. This observation is not intended as a rhetorical point. It reflects a genuine structural alternative: the physical phenomena that appear to require extra dimensions from one mathematical starting point may require no extra dimensions at all from another.

We close by emphasising that this comparison is offered in a spirit of mutual engagement, not refutation. Pinčák et al.'s framework and Category τ are both serious attempts to address some of the deepest problems in theoretical physics. The fact that they reach structurally different conclusions from different premises is itself a valuable datum for the field. It suggests that the ultimate experimental discrimination between these approaches—whether through gravitational-wave observations of QNM spectra, precision tests of Newton's law at sub-millimetre scales, or future collider searches for KK states—will be among the most consequential tests physics can design.

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